TOWARDS A LOW-ORDER DYNAMIC STALL MODEL USING A
PARAMETRIC PROPER ORTHOGONAL DECOMPOSITION

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Abstract
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Measured unsteady surface pressures, which are a function of both space and time, of a harmonically pitching airfoil are expressed in terms of a parametric proper orthogonal decomposition (PPOD) in order to obtain an optimum (in the mean-square sense) modal representation. This decomposition is formulated in such a way that the resulting spatial modes act optimally over the entirety of a parameter space defined by the airfoil pitching motion characteristics, i.e. for attached flow pitching, light stall, and deep stall. This method provides a systematic and quantitative framework by which to elucidate common and disparate features of the light and deep dynamic stall processes and provides a bridge to the development of low-order models for the prediction of unsteady airloads, such as the normal force and quarter-chord pitching moment.

This work primarily focuses on the development of two low-order models, distinguished by frame of reference, used for the reconstruction of unsteady aerodynamic loads. The first model decomposes the unsteady pressure field where the steady inviscid pressure field, provided by a Smith-Hess panel method, is removed. Conversely, the second model decomposes the unsteady pressure field with the fully viscous, steady pressure field removed. In each model, the parameter-independent modal shapes are determined from unsteady surface pressures of an arbitrarily chosen ref-
ference airfoil geometry operating over a large range of pitching trajectories. It is shown that the aerodynamic loads of the reference geometry are reconstructed with as few as 5 PPOD modes. For the first model, the airloads of a candidate airfoil, one where the unsteady surface pressure field is desired for a given pitching trajectory, are shown to be reconstructed using the same 5 reference PPOD modes plus an additional spatial mode calculated from the candidate airfoil’s steady pressure field. Likewise, the second model is capable of reconstructing the candidate airfoil’s unsteady surface pressure with only the 5 reference PPOD modes. A subtle result realized by comparing the two models is that the nature of the dynamic stall process is a general feature acting as a perturbation to the steady behavior of a given airfoil geometry.
For Lauren, Carter, and Caleb
CONTENTS

FIGURES ................................................................. vi
TABLES ................................................................ ix
ACKNOWLEDGMENTS .................................................. x
SYMBOLS .................................................................. xii

CHAPTER 1: INTRODUCTION ........................................... 1
  1.1 The problem ...................................................... 1
  1.2 Origin of dynamic stall in the rotor environment .......... 2
  1.3 Overview of dynamic stall ....................................... 5
    1.3.1 Stall regimes ............................................... 6
    1.3.2 Parameters important to dynamic stall analysis ......... 8
  1.4 Unsteady pitching aerodynamic load modeling .......... 10
    1.4.1 Theoretical modeling review ............................ 10
    1.4.2 Semi-empirical models .................................. 12
  1.5 Objective ......................................................... 14

CHAPTER 2: EXPERIMENTAL PROCEDURE ..................... 15
  2.1 Wind tunnel ...................................................... 16
  2.2 Test section ...................................................... 16
  2.3 Pitching mechanism ............................................ 19
  2.4 Airfoils ............................................................ 20
  2.5 Sensors ........................................................... 23
    2.5.1 Surface pressure ........................................... 23
      2.5.1.1 Kulite transducers ................................. 24
      2.5.1.2 Endevco transducers ............................. 25
    2.5.2 Freestream pressure ....................................... 26
    2.5.3 Angle of attack .......................................... 27
  2.6 Calibration ....................................................... 28
    2.6.1 Airfoil surface pressure .................................. 28
    2.6.2 Test section calibration .................................. 28
  2.7 Data acquisition ................................................ 29
  2.8 Aerodynamic load calculation ................................ 30
FIGURES

1.1 Rotor environment: forward flight ............................................. 4
1.2 Location of dynamic stall events [2] ........................................... 5
1.3 Dynamic stall chronology ............................................................ 7
1.4 Flow behavior in the light (top) and deep (bottom) stall regimes [41] 8
2.1 Whitefield Mach 0.6 wind tunnel: top view .................................. 17
2.2 Dynamic stall test section: exploded view .................................... 18
2.3 Pitching mechanism detail .......................................................... 20
2.4 Vector diagram of mechanism linkage system ............................... 21
2.5 Exploded view of NACA 23012 experimental model ...................... 22
2.6 NACA 23012 with sensors and packaging .................................... 23
2.7 Size of leading edge Kulite (XCL-062) ......................................... 25
2.8 Size of trailing edge Endevco (8515C-15) and wiring ..................... 26
2.9 Wire-side angle of attack sensor connection .................................. 27
2.10 Dynamic calibration with linear fit .............................................. 29
2.11 Test section calibration setup ..................................................... 30
2.12 Airfoil coordinate systems .......................................................... 31
3.1 Model schematic for “star” reference frame ................................. 34
3.2 Model schematic for “hat” reference frame ................................... 35
4.1 Comparison of NACA 23012 static $-C_p$ at $\alpha = 9.8^\circ$ .............. 44
4.2 Comparison of NACA 23012 static $C_n$ ....................................... 45
4.3 PA2 $\phi$-mode eigenvalues, $\lambda_i$ as a function of mean angle of attack .. 48
4.4 PA2 $\phi$-modes ................................................................. 49
4.5 PA2 $\phi$-mode coefficients, $a_i$ ................................................... 50
4.6 PA2 $\phi$-mode normal force, $C_n^*$, reconstruction ........................ 51
4.7 PA2 $\phi$-mode quarter-chord moment, $C_m^*$, reconstruction ............ 52
4.8 PA2 $\phi$-mode aerodynamic load reconstruction error convergence ... 54
6.4 PA2 “star” method $d_1$ maximum amplitude with projected values from the PA1 and NACA 23012 ................................................. 99
6.5 PA2 “star” method $d_2$ maximum amplitude ........................................ 100
6.6 PA2 “star” method $d_2$ minimum amplitude .......................................... 101
6.7 PA2 “star” method $d_3$ maximum amplitude .......................................... 102
6.8 PA2 “hat” method $d_1$ minimum amplitude .......................................... 104
6.9 PA2 “hat” method $t/T(d_1|_{\min})$ values ............................................... 104
6.10 PA2 “hat” method $d_2$ minimum amplitude ......................................... 105
6.11 PA2 “hat” method $d_3$ maximum amplitude ......................................... 106

B.1 NACA 23012 normal force reconstruction, $C_n^*$, from projected $\psi$-mode 113
B.2 NACA 23012 quarter-chord moment reconstruction, $C_m^*$, from projected $\psi$-mode .................................................. 114
B.3 NACA 23012 first split mode, $\theta_1$, and coefficient, $g_1$ ..................... 115
B.4 NACA 23012 aerodynamic loads reconstructions from $\psi$-modes plus the first split mode, $\theta_1$ ........................................... 115
B.5 NACA 23012 first split mode, $\theta_1$, and coefficient, $g_1$ using steady pressure field projection ............................................. 116
B.6 NACA 23012 aerodynamic loads reconstructions from $\psi$-modes plus the first split mode, $\theta_1$ calculated from the steady pressure field . . . 116
TABLES

2.1 Airfoil characteristics ........................................ 21
2.2 NACA 23012 pressure port details .......................... 24
4.1 PA2 parameter cases for “star” method ($\alpha_1 = 8^\circ$ for all cases) .... 47
5.1 PA2 parameter cases for “hat” method ($\alpha_1 = 8^\circ$ for all cases) .... 76
6.1 PA2 $\psi$-MODE SHAPE CONTRIBUTIONS .................. 95
A.1 PA1 pressure port details ..................................... 111
A.2 PA2 pressure port details ..................................... 112
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and longer nights of work. Our office has consistently been filled with laughter and thoughtful conversation because of these two, and I count myself very fortunate to have had the opportunity to work alongside them.
# SYMBOLS

**English**

- \(a\) Modal coefficient
- \(a_\infty\) Freestream speed of sound, \(\text{m/s}\)
- \(b\) Semi-chord length, \(c/2\)
- \(c\) Chord length, m
- \(C_a\) Non-dimensional axial force, \(A/qc\)
- \(C_d\) Non-dimensional drag force, \(D/qc\)
- \(C_l\) Non-dimensional lift force, \(L/qc\)
- \(C_n\) Non-dimensional normal force, \(N/qc\)
- \(C_m\) Non-dimensional quarter-chord pitching moment, \(M/qc\)
- \(C_p\) Non-dimensional pressure, \(P/q\)
- \(d\) Modal coefficient
- \(f\) Pitch frequency, Hz
- \(g\) Modal coefficient
- \(k\) Reduced frequency, \(\pi f c/U_\infty\)
- \(M_\infty\) Freestream Mach number, \(U_\infty/a_\infty\)
- \(n\) Integer dummy variable
- \(p\) Modal coefficient
- \(P\) Pressure, psi
- \(P_0\) Total pressure, psi
- \(P_S\) Static pressure, psi
- \(q\) Freestream dynamic pressure, \(1/2 \rho_\infty U_\infty^2\)
- \(r\) Radial location, m
- \(R\) Autocorrelation function; Total length of rotor blade, m
- \(Re_c\) Reynolds number based on airfoil chord, \(U_\infty c/\nu\)
- \(s\) Span length, m
- \(t\) Time, s; Thickness, m
- \(u'\) Fluctuating velocity, m/s
- \(U_\infty\) Freestream flow speed, m/s
- \(U_\parallel\) Rotor in-plane velocity, m/s
- \(U_⊥\) Rotor plane normal velocity, m/s
- \(x\) Chord-wise location, m

**Greek**

- \(\alpha\) Angle of attack, deg
- \(\dot{\alpha}\) Pitch rate, deg/s
$\alpha_0$  Mean angle of attack, deg
$\alpha_1$  Oscillation amplitude, deg
$\gamma$  Specific heat ratio
$\epsilon$  “Star” reference frame scaling factor
$\theta$  Split mode shape
$\lambda$  Proper orthogonal decomposition eigenvalue
$\mu$  Advance ratio
$\nu$  Kinematic viscosity, $m^2/s$
$\xi$  Non-dimensional chord-wise coordinate, $x/c$
$\rho$  Density, $kg/m^3$
$\phi$  Parametric proper orthogonal decomposition mode shape
$\psi$  Rotor azimuthal angle, deg; Parametric proper orthogonal decomposition mode shape
$\Omega$  Rotor angular velocity, rad/s
$\omega$  Weighting function; Pitch angular frequency, rad/s

**Subscript**
- $cg$  Center of gravity
- $CT$  Candidate airfoil
- $i$  Integer index variable
- $j$  Integer index variable
- $REF$  Reference airfoil
- $sp$  Stall penetration
- $ss$  Static stall
- $T$  Number of time stamps
- $x$  Number of spatial coordinates
- $\infty$  Freestream

**Superscript**
- $\tilde{}$  Split mode difference
- $^*$  Denotes “star” method
- $^\wedge$  Denotes “hat” method
CHAPTER 1

INTRODUCTION

1.1 The problem

Because of its long-standing influence on the rotorcraft, wind energy, and turbomachinery communities, the problem of dynamic stall continues to cultivate new investigations concerning the underlying physical mechanisms of the process. These are motivated by attempts to better understand, predict, and control its aerodynamic influence on airfoils. This flow phenomenon is characterized as the unsteady separation evolution in which stall is delayed as a lifting surface rapidly exceeds its static stall angle of attack. Associated with such a motion trajectory is the initiation, growth, and convection of a large, coherent, vortical structure near the leading edge of the surface known as the dynamic stall vortex (DSV). The emergence of this flow structure introduces large fluctuations in the pressure field and, consequently, develops nonlinear aerodynamic loadings, exhibiting large excursions from their respective static counterparts with potential to be detrimental to the mechanical system.

Despite being the topic of an extensive body of research spanning the past six decades, the dynamic stall process remains to be fully understood and accurately predicted for arbitrary airfoil motions. Ericsson and Reding stated that, “only if the unsteady stall mechanism is understood can an ‘analytic extrapolation’ to full scale [rotors] be made with confidence,” [12] conveying the importance and difficulty of predicting the unsteady process. Historically, theoretical modeling of unsteady aerodynamics is limited to attached flow cases such as the classical problems contributed by Theodorsen [50], Wagner [53], and von Kármán and Sears [27]. Because
of the difficulties found in accurately and consistently predicting the stall process, engineering efforts for the design of advanced unsteady airfoil sections rely heavily on experimentally driven, semi-empirical models. Typical models, like the Leishman-Beddoes [30] and the ONERA [42], tune their model-specific system coefficients and time parameters through the acquisition of experimental results for the steady and dynamic pitching of the airfoil under investigation. These models have proven to be computationally faster and cheaper than full computational fluid dynamics (CFD) simulations[52]. However, they require the experimental data they are attempting to predict, or at the very least, minimal unsteady testing for all airfoils considered. In this sense, the models are postdictive. To continue the advancement of rotor blade technology, it is desirable to have a dynamic stall model that is computationally fast, robust, and requires very few input parameters for the airfoil to be analyzed.

1.2 Origin of dynamic stall in the rotor environment

Compared to fixed wing aircraft, which produce lift by propelling their wings in the forward direction, a helicopter achieves lift by rotating its rotor blades in a plane parallel to the ground. Because of the rotation, the local flow velocity seen by the blade is dependent on radial location. When the vehicle begins to move in forward flight, the local flow velocity is further complicated by also becoming a function of azimuthal position in the rotor plane given as

\[ U(r, \psi) = \Omega r + U_{\infty} \sin(\psi), \]  

(1.1)

where \( \Omega \) is the rotor angular velocity, \( r \) is the radial location, \( U_{\infty} \) is the forward flight speed, and \( \psi \) is the azimutal angle defined in Figure 1.1. The sinusoidal variation in dynamic pressure, causing an imbalance of lift across the vehicle if unaccounted for, is compensated for by pitching the rotor blades to higher angles of attack on the
The retreating side of the rotor plane. Figure 1.1 schematically depicts the variation of the local flow velocity in forward flight. A non-dimensional parameter used to relate the rotational speed of the rotors to the forward flight speed is the advance ratio defined as

$$\mu \equiv \frac{U_\infty}{\Omega R},$$

where $R$ is the total length of the rotor. More specifically, the advance ratio compares the tip speed of the rotor to the forward flight speed. Another complication is the growth of a reversed flow region as the forward flight speed increases. This region further inhibits the retreating side of the rotor plane from producing lift. Using the advance ratio and tip speed to non-dimensionalize Eq. 1.1, the reversed flow region can be defined as existing in the following radial positions:

$$\frac{U(r, \psi)}{\Omega r} = \frac{r}{R} + \mu \sin(\psi) < 0,$$

$$r < -R\mu \sin(\psi).$$

When $\psi \in [180^\circ, 360^\circ]$ the right-hand side of Eq. 1.4 is positive valued suggesting that the reversed flow region is a circle of radius $r_{\text{reverse}} = R\mu$ centered at $(r = R\mu/2, \psi = 270^\circ)$. Clearly, once the flight speed reaches an advance ratio $\mu = 1$ the reversed flow region has a radius extending the full length of the rotor. To accommodate the lift deficit associated with this region, rotor blades are required to pitch to even higher angles of attack on the retreating side. Once the static stall angle of attack has been exceeded, the rapid cyclic variation known as “1/rev” pitching becomes the basis for dynamic stall inception.

While forward flight is one means to its inception, dynamic stall can also occur during high blade loading maneuvers. After an investigation of data from the UH-
60A Airloads Program, Bousman [2] located the existence of dynamic stall events in several regions of the rotor plane, shown in Figure 1.2. The azimuthal positions of the localized events suggest that the study and prediction of dynamic stall should consider a range of chord-based Reynolds ($Re_c$) and Mach ($M_\infty$) numbers.

Figure 1.1. Rotor environment: forward flight
1.3 Overview of dynamic stall

McCroskey [35, 37], Carr et al [4, 5], and Leishman [29] have all presented detailed accounts of the dynamic stall phenomenon. Likewise, it would seem appropriate to review the current understandings of the dynamic stall phenomenology and its theoretical development. Figure 1.3 depicts the most generic flow field characteristics and their respective aerodynamic loads during a typical dynamic stall process. The chronology of the flow field during the angle of attack trajectory can be described as follows:

1. As the upstroke of the angle of attack trajectory begins, the flow over the upper
surface of the airfoil remains attached in excess of the static stall angle of attack. During this time, however, there can exist flow reversal without the separation of the boundary layer due to unsteady effects. That is, the external flow field continues to follow a path described by the airfoil profile despite an upstream movement of flow contained within a thin layer near the surface.

2. Boundary layer vorticity begins to accumulate near the leading edge after surpassing the static stall limit.

3. Once the amount of vorticity has reached some saturation limit for the boundary layer, an eruption into the external flow occurs causing the subsequent breakdown of the boundary layer. This eruption forces a high volume of vorticity into the external flow, inducing a viscous-inviscid interaction and the roll-up of the most salient feature of the process — the dynamic stall vortex. The coalesced vortex then grows in strength, and its presence as a localized low pressure disturbance on the surface causes an increase in the lift curve slope. However, the aft convection of the vortex causes the airfoil to stall in moment loading, consequently producing the large, and often detrimental, nose-down pitch moment. Maximum values of the lift and nose-down pitch moment occur as the vortex reaches the trailing edge of the airfoil.

4. The vortex is then ejected into the trailing wake invoking the full separation of the boundary layer over the entirety of the upper surface of the airfoil and the initiation of lift stall.

5. During the downstroke of the angle of attack trajectory, the flow begins to reattach from leading edge to trailing edge.

1.3.1 Stall regimes

The dynamic stall process is often categorized into stall regimes to describe the order of magnitude of the resulting separated region. These regimes are defined by the development of the dynamic stall vortex with respect to the motion of the airfoil
Figure 1.3. Dynamic stall chronology

and are schematically depicted in Figure 1.4. When the airfoil begins to pitch down before the vortex has fully developed, it detaches from the surface prematurely. This underdeveloped vortex detachment creates a separation region on the order of the airfoil thickness and is termed light stall. While only minor penalties are experienced in the moment and drag loads for this regime, unsteady lag effects are still significant and can be difficult to model. If, on the other hand, the vortex is allowed to fully mature and detach from the airfoil surface while the airfoil is still pitching upwards, a separation region on the order of the airfoil chord is created [41]. This regime is termed deep stall and introduces the most detrimental aerodynamic loads experienced in the pitching airfoil environment. An example of the air loads created by this regime

7
are shown in Figure 1.3.

Figure 1.4. Flow behavior in the light (top) and deep (bottom) stall regimes [41]

1.3.2 Parameters important to dynamic stall analysis

Previous dynamic stall experiments have been dedicated to determining the sensitivity of the phenomenon to flow and motion parameters [35, 37]. The most relevant parameters, which will also be considered in this dissertation, have been found to be

- reduced frequency, $k$,
• chord-based Reynolds number, $Re_c$,
• Mach number, $M_\infty$,
• mean angle of attack, $\alpha_0$, and
• oscillation amplitude, $\alpha_1$.

The reduced frequency is defined as

$$k = \frac{\omega c}{2U_\infty} = \frac{\pi f c}{U_\infty},$$

where $\omega$ and $f$ are the angular and linear pitching frequency, respectively. This parameter is a ratio of the unsteady pitching time scale to the free stream velocity time scale. In a basic sense, it is the number of oscillations in the time it takes the airfoil to travel one semi-chord, $c/2$. Ultimately, reduced frequency is a measure of the unsteadiness of the flow introduced by the forced pitching of the airfoil. Values below $k = 0.05$ are considered to be quasi-steady, while $k \geq 0.05$ is considered unsteady. During flight, the rotor blade sees a multitude of reduced frequencies, as it is a function of radial position and azimuthal location, similar to the local velocity.

Chord-based Reynolds number is useful in defining the boundary layer characteristics present on the rotor. Here, it is defined as

$$Re_c = \frac{cU_\infty}{\nu},$$

for a given chord length $c$, free stream velocity $U_\infty$, and kinematic viscosity of the fluid (air) $\nu$. Effects of Reynolds number have typically been determined to be small for dynamic stall \[35\], at least in regimes relevant to rotor craft applications.

The mean angle of attack, $\alpha_0$, and the oscillation amplitude, $\alpha_1$, have a significant effect on the dynamic stall process. Together, these two parameters define the stall regimes previously described \textit{i.e.} unsteady attached flow, stall onset, light stall, and deep stall. The oscillation amplitude and pitching frequency are also used to define
parameters such as pitch rate, which is useful when describing the state of motion of
the airfoil.

1.4 Unsteady pitching aerodynamic load modeling

In order to continue the advancement of rotor blade technology, the dynamic stall
process must be understood and reliably predicted. To this end, many theoretical
approaches have been employed; however, the chaotic nature of the phenomenon has
limited the ability and application of such methods. Because of this shortcoming,
current technology uses semi-empirical models. To provide a basis for this work, both
methodologies will be briefly reviewed.

1.4.1 Theoretical modeling review

Theoretical work in attached flow unsteady aerodynamics began in the 1920s
with the work of Prandtl when he formulated the time dependent circulation of a
periodically pitching airfoil [23]. Stemming from this work, Wagner [53], in 1924,
considered the evolution of lift about an airfoil at a fixed angle of attack impulsively
started from rest to a fixed uniform velocity in an inviscid, incompressible fluid. The
problem was solved for a two-dimensional flat plate using conformal mapping. His
result is a function, \( \phi(s) \), describing the lift as a function of non-dimensional time
\( s = U_\infty t/b \), which is the amount of time for the flow to travel one semi-chord \( (b = c/2) \). At
\( s = 0 \) the lift is exactly half of the steady state value and asymptotically approaches
this value as time progresses. The utility of this result is that the aerodynamic loads
for any arbitrary motion are computed by superposition. The Leishman-Beddoes
model uses this function to model attached flow unsteady pitching.

Following the work of Wagner, in 1935, Theodorsen also considered the prediction
of the aerodynamic forces on an oscillating thin airfoil. His formulation determined
a frequency domain solution for the lift and moment of a flat plate performing sim-
ple harmonic oscillations in pitch and plunge within a potential flow field \[50\]. The airfoil and infinite wake are both modeled as planar vortex sheets with the wake being composed of vorticity shed from the airfoil trailing edge, traveling downstream at the freestream flow speed. Theodorsen ultimately describes the non-dimensional aerodynamic loads in terms of steady components and unsteady components, which are dependent on the reduced frequency, \(k\), of the motion. Perhaps the most notable result of this work was the development of a transfer function describing the relationship between the oscillation frequency and the unsteady component of the aerodynamic loadings. This transfer function is simply known as the Theodorsen function, \(C(k)\), where \(k\) is the reduced frequency of the oscillation. Theodorsen’s groundbreaking work is still used for the analysis of many unsteady aerodynamic problems. However, it must be noted that this theory is restricted to attached flow modeling due to the assumptions made in its derivation.

After Theodorsen, von Kármán and Sears approached the same problem from a more physical position in an attempt to “eliminate unnecessary mathematical complications and to try to use only the basic conceptions of the vortex theory familiar to the modern aeronautical engineer.” \[27\] They were concerned that the formulations of the past by Birnbaum, Wagner, Küssner, Glauert, and Theodorsen were overly complicated and were difficult to understand when considering their physical significance. For this reason, von Kármán and Sears formed a theoretical model for unsteady airfoil characteristics relying on basic vortex theory. This model, like those before, represents the airfoil as a vortex sheet and the wake as an accumulation of deposited vorticity resulting from the shed airfoil circulation. The strength of wake vortices is determined by maintaining a conservation of total circulation of the system using Kelvin’s theorem. Wake vortices were also assumed to move with the fluid and to lie in a flat plane, resulting in the restriction of the airfoil to small motions. Through the application of conformal mapping and thin airfoil theory, the
authors assert that the lift and moment values of an airfoil undergoing a prescribed non-uniform motion consist of a combination of three effects:

1. the quasi-steady loading,
2. the contribution due to an apparent mass, and
3. the contribution due explicitly to the vorticity distribution found in the wake.

This formulation offers a simple theoretical approach to generalized airfoil motion in a concise physical format and produces the same results as those found previously.

In their recent approach, Ramesh et al. [44] develop an inviscid theoretical method to deal with arbitrary airfoil motions involving large amplitudes. This method implements the use of a non-planar wake and disregards any small-angle approximations. Unsteady boundary conditions of the airfoil are satisfied using a vorticity distribution, similar to thin airfoil theory. By tracking the shed vorticity in an inertial frame of reference and calculating the local downwash in a body frame of reference, the lift and drag are determined at each time step. The authors note the following advantages over the previously described classical theory:

- Because there are no small-angle approximations, the airfoil normal and axial forces can be calculated, as opposed to just normal force.
- The inclusion of a non-planar wake allows for a more accurate description of the aerodynamic loading, especially at higher rates of motion.
- Any arbitrary motion can be considered.
- The theory allows for the inclusion of viscous effects and even the modeling of leading edge vortex (LEV) evolution.

1.4.2 Semi-empirical models

The most notable semi-empirical dynamic stall model used throughout the rotorcraft industry is the Leishman-Beddoes model [15], [21], [30]. With an attempt at maintaining a physical representation of the flow physics, this model is composed of four subsystems:
1. attached flow pitching (Duhamel superposition),
2. a separated flow model,
3. a dynamic stall onset model, and
4. a dynamic stall model for vortex induced aerodynamics.

The complete model operates as an open-loop system, with the output of each subsystem acting as the input for the next. The premise of the model is to use analytical assumptions for each subsystem, i.e. indicial response (similar to Wagner’s function) for attached flow pitching, and adjust their responses using empirical coefficients. Most of these coefficients may be described from the steady airfoil characterization, however, four are left to be derived from unsteady pitching data. One advantage of the model is that it can predict the unsteady airloads for arbitrary motion trajectories due to its superposition nature. Numerous studies [46–49] have been conducted dealing with the adjustment of model parameters, such as stall criterion, to better suit the flow field or airfoil geometry being considered.

Another widely used semi-empirical model is the ONERA dynamic stall model [42]. This approach uses a set of empirically-derived nonlinear ordinary differential equations as a basis for calculating the unsteady aerodynamic loads. The inviscid, attached flow is described by a first-order differential equation, while the nonlinear viscous effects introduced by stall are represented by a second-order differential equation. The order of each differential equation was determined empirically to best describe the experimental behavior. Similar to the Leishman-Beddoes model, the model coefficients are determined from experimental data, however, this model relies much more heavily on parameter identification from oscillating airfoil experiments.

While other semi-empirical models exist, most are some derivative or an addendum of the previously described two. Comparison studies of these models have lead to the conclusions that a single model cannot be claimed as best, only better than the others in a given regime [24]. These models also leave much to be desired due
to their requirement of unsteady airfoil data to appropriately tune their parameters for each geometry considered. Ideally, the semi-empirical model would be capable of handling arbitrary motion and changes in geometry without the expensive overhead of the unsteady testing of each airfoil to be considered.

1.5 Objective

In light of these shortcomings, this work will demonstrate the development of a low-order dynamic stall model capable of predicting the unsteady spatial pressure field behavior of airfoil geometries from only their static characterization, given the unsteady characterization of a single, arbitrary reference geometry. A model of this type is desirable because the static characterization of airfoils is much less expensive and time-consuming than the acquisition of even a minimal amount of unsteady data. This model is developed using parametric proper orthogonal decomposition (PPOD) [10, 14] and split-mode proper orthogonal decomposition [3]. Its formulation not only allows the unsteady surface pressure field of an airfoil to be predicted, but also provides a framework for additional in-depth analysis of the dynamic stall phenomenon. The remainder of this dissertation is organized as follows:

- the experimental facility and setup is described in Chapter 2,
- an overview of the model from two different frames of reference and a review of the methods used are presented in Chapter 3,
- Chapter 4 demonstrates the calculation of model basis functions from the reference airfoil data using what will be called the “star” reference frame,
- Chapter 5 calculates the model basis functions from reference airfoil data using another framework called the “hat” reference frame,
- physical interpretations of the resulting mode shapes from each reference frame are discussed in Chapter 6 and
- finally, conclusions and suggestions for future work are given in Chapter 7.
CHAPTER 2

EXPERIMENTAL PROCEDURE

During forward flight, a helicopter rotor sees velocity fluctuations from three contributing sources: in-plane velocity ($U_\parallel$), plane normal velocity ($U_\perp$), and angle of attack ($\alpha$). In-plane fluctuations are a product of 1/rev motion (rotor rpm), radial position, and heaving motion introduced by lead-lag hinges. Downwash from tip vortices and rotor wake interaction, aeroelastic effects, and plunging motion due to flap hinges create plane normal fluctuations. The angle of attack is a combination of aeroelastic effects and 1/rev pitching, which is a direct input from the pilot.

Most dynamic stall experiments found in literature have been conducted by considering only the effect of angle of attack variation in 2D flow fields, e.g. [6, 8, 16-20, 36, 38, 40]. However, a small number of studies [7, 13, 31, 34, 51] have assessed the contribution of $U_\perp$ by exposing 2D airfoil sections to plunging motions. It was found that these motions produced similar load hysteresis values as those found in pitching experiments. Pierce et al. [43] investigated the effects of $U_\parallel$ on the dynamic stall process by varying the freestream velocity. This study concluded that the freestream oscillations effected the aerodynamic moment of the unsteady airfoil when pitched near static stall, however, the aerodynamic work done on the airfoil was rather unchanged at all angles. McCroskey et al. [39] explored the effects produced by a 3D flow field, by instrumenting a model rotor with absolute pressure transducers and skin friction gages. One of the most important conclusions from this study, as far as experimental techniques are concerned, was that at the onset of dynamic stall, 3D effects do not significantly alter the unsteady airfoil loads, as compared to those
found in 2D pitching tests. Bousman [2] discovered similar results when comparing loads from the UH-60A Airloads Program stating that the dynamic stall seen on the in-flight rotor was essentially the same as that observed in 2D pitching wind tunnel studies. The results from both 3D investigations suggest that variation of the airfoil angle of attack is the dominant driver in the dynamic stall process and confirm the validity of 2D flow fields for the analysis of unsteady airfoil loads. As such, all experiments considered in the current study have been conducted assuming only angle of attack variations through pitching motions in a 2D flow field.

2.1 Wind tunnel

All experimental investigations to-date were completed in the Mach 0.6 closed-return wind tunnel located in the Notre Dame Hessert Laboratory at Whitefield. A top-down view of the wind tunnel is shown in Figure ???. The flow is driven by a 1750 h.p. variable rpm AC motor that turns an 8 ft diameter, 2-stage, high-solidity fan. At a given flow speed, equilibrium temperature of the tunnel is achieved by actively cooling the turning vanes downstream of the fan via 40°F water supplied by a 125 ton chiller coupled with a 1000 ton-hr ice storage system. The flow is then conditioned by a 5 in. thick honeycomb wall (0.25 in. nominal diameter) followed by five #28 wire screens leading to a 6:1 contraction. The resulting turbulence intensity in the test section is $u'/U_\infty \approx 0.01$ or 1% [11], which is comparable to free-flight conditions.

2.2 Test section

The tunnel has three interchangeable test sections, of which one has been retrofitted to accommodate unsteady, pitching airfoil experiments. An exploded view of the test section is depicted in Figure 2.2. Airfoils are mounted between two 0.75 in. thick aluminum splitter plates and fixed to rotate about their quarter-chord position. Each
Figure 2.1. Whitefield Mach 0.6 wind tunnel: top view

side of the airfoil is fitted with a 0.5 in. thick Lexan end plate (rotating disk), which is lubricated with synthetic grease, housed inside the splitter plate, and secured with an aluminum enclosure bolted to the splitter plate, creating a pseudo-labyrinth seal. The aluminum enclosure is backed with Teflon stripping to reduce contact friction with the rotating disk. Instrumentation is carried out of the test section opposite the pitching mechanism through the wire shaft, while the pitching motion is driven through a hardened steel shaft called the torque tube, connected to the mechanism. The torque tube and wire shaft are both supported by passing through ball bearings housed in aluminum casings, secured to the test section floor, found just outside of each splitter plate. The bearing housings facilitate the transmission of aerodynamic loads to the test section. Attaching the airfoil to the system are two airfoil connectors. On the torque tube side, the connector is made of hardened steel, while the
The wire side connector is aluminum. Hardened steel pins attach the connectors through the end plates to the airfoil, which are secured at the airfoil using 4-40 set screws. The wire shaft connects to the wire side airfoil connector via set screws. However, the torque side connector attaches to the torque tube through a six-sided parallel key spline, facilitating the transmission of the pitching mechanism input. The torque tube connects to the pitching mechanism via a 100 tooth involute spline. Both the torque tube and wire shaft exit the wind tunnel through plexiglass windows fitted with low-friction bushings.

Figure 2.2. Dynamic stall test section: exploded view
2.3 Pitching mechanism

The foundation of the current dynamic stall facility is the 2 degree-of-freedom pitching mechanism displayed in Figure 2.3. The mechanism is a dual-input walking beam design capable of producing single frequency or dual frequency output signals of the form

\[ \alpha(t) = \alpha_0 + \alpha_1 \sin(\omega_1 t + \phi_1) + \alpha_2 \sin(\omega_2 t + \phi_2), \]  

where \( \alpha(t) \) is the instantaneous angle of attack of the airfoil, \( \alpha_0 \) is the mean angle of attack, \( \alpha_1 \) is the 1/rev pitch amplitude, \( \alpha_2 \) is the \( n/\text{rev} \) pitch amplitude, \( \omega_1 \) and \( \phi_1 \) are the 1/rev circular frequency and phase, respectively, and \( \omega_2 \) and \( \phi_2 \) are the \( n/\text{rev} \) circular frequency and phase, respectively. The adjustable pitch link is used to control the mean angle of attack, \( \alpha_0 \), by varying its length. Oscillation amplitude provided by each flywheel input is controlled by offsetting the respective spindle placement from the center using the Acme threads. The walking beam acts as a mechanical adder of the two frequency and amplitude inputs from the two flywheels, which are attached via the push rods. The end result is a vertical displacement of the pitch link and the subsequent rotation of the pitch horn to produce complex pitching trajectories at the torque tube connection. A vector diagram demonstrating the production of the pitch motion is shown in Figure 2.4.

Each flywheel is driven by a Marathon 10 hp Black Magic 420 VAC motor. Dynapar internal encoders monitored by Yaskawa F-7 drives provide rpm control independently for each motor. The pair of F-7 control drives can either be prescribed a specific frequency digitally via the controller’s keypad or given a variable analog voltage (0–10 VDC) with a dial potentiometer to control motor rpm. In this remote setting, motor frequency scales linearly with voltage, where 0–30 Hz of physical pitching corresponds to 0–10 V on the input. The motors connect to their respec-
tive drive shafts through Duraflex flexible couplings. A Dodge Imperial pillow block aligns and fixes each drive shaft to the test stand. The pillow blocks are bolted to a 2 in. thick steel plate found at the top of the test stand. Each flywheel attaches to its respective drive shaft with a flange connection. The 1/rev flywheel measures 12 in. in diameter, while the \( n/\text{rev} \) flywheel is 10 in. in diameter.

2.4 Airfoils

For this dissertation, three different airfoil test articles were constructed. The airfoil geometries considered are a NACA 23012 and two proprietary designs. Details of the airfoil characteristics are given in Table 2.1. The design, construction, and instrumentation procedures for each airfoil model are nearly identical, therefore, only the process for the NACA 23012 will be described in detail.

As defined by the definition for a 5-digit NACA airfoil section, this airfoil has
Figure 2.4. Vector diagram of mechanism linkage system

Table 2.1

<table>
<thead>
<tr>
<th>Airfoil</th>
<th>L.E. radius, (r_{le}/c)</th>
<th>Thickness, (t/c)</th>
<th>L.E. modifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>NACA 23012</td>
<td>0.01581</td>
<td>0.1200</td>
<td>–</td>
</tr>
<tr>
<td>PA1</td>
<td>0.00589</td>
<td>0.0998</td>
<td>tripped (thin paint line)</td>
</tr>
<tr>
<td>PA2</td>
<td>0.00987</td>
<td>0.1143</td>
<td>tripped (thin paint line)</td>
</tr>
</tbody>
</table>

A design lift coefficient of 0.3 in steady flight. Its maximum camber is located at \(x/c = 0.15\) and has a maximum thickness ratio of \(t/c = 0.12\). The steady flight characteristics of this airfoil at several Reynolds numbers are well documented by Abbott and von Doenhoff [1]. The 3-piece, “clam shell” design is fabricated from high strength Aluminum (Al 7075-t6) cut on a 4-axis CNC mill. Figure 2.5 illustrates
the components of the airfoil.

This design facilitates the proper installation of pressure transducers and safe storage of transducer cables while maintaining a rigid housing with hydrodynamically smooth surfaces (Figure 2.6). The leading edge connects to the main body via eight 8-32 cap screws. Likewise, the lower surface secures to the assembly with eighteen 8-32 cap screws. Two large spanwise sections are removed from the interior mass in attempt to move the center of gravity closer to the pitching axis and lower the inertia of the system. The resulting center of gravity resides at $x/c_{cg} = 0.425$. The model chord length is set at $c = 10$ in. and has a span of $s = 14$ in. to operate within the splitter plates.

Surface pressure is measured along the midspan using 31 high-frequency-response absolute pressure transducers (33 for the proprietary designs). The mounting locations and transducer types for the NACA 23012 are detailed in Table 2.2. Similarly, the port locations and types for the proprietary geometries are located in Appendix A. The transducers are positioned along the chord, at the midspan, in a cosine distribution described as

![Figure 2.5. Exploded view of NACA 23012 experimental model](image)
Figure 2.6. NACA 23012 with sensors and packaging

\[ x_n/c = 1 - \cos \left( \frac{n\pi}{2N} \right), \quad (2.2) \]

where \( N = 15 \), \( n = 0, 1, ..., N - 1 \), and \( x_n/c \) is the non-dimensional chord station of the pressure transducer. The last pressure port is located at \( x_{15}/c = 0.975 \). Each port location aft of 60% chord has a wire track removed from the surface in order to provide access to the inside of the airfoil. Here, the sensor wires are glued inside of the track, covered with J.B. Weld, and sanded smooth to the surface. All sensors forward of 60% are surface mounted through the airfoil skin and backed with RTV to hold each in position.

2.5 Sensors

2.5.1 Surface pressure

Because of the unsteady nature of the experiment, high-frequency-response, absolute pressure transducers are required to resolve the time-varying aerodynamic loads. For this airfoil two types of transducers were used: cylindrical transducers (Kulite) and flat, surface mount transducers (Endevco).
TABLE 2.2

NACA 23012 PRESSURE PORT DETAILS

<table>
<thead>
<tr>
<th>Chord Station</th>
<th>Pressure Sensor</th>
<th>#</th>
<th>x_n/c</th>
<th>Manufacturer</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td>0</td>
<td>0.0000</td>
<td>Kulite</td>
<td>XCL-062-25A</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>1</td>
<td>0.0055</td>
<td>Kulite</td>
<td>XCL-062-25A</td>
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<td>2</td>
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<td>2</td>
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<td>XCL-062-25A</td>
</tr>
<tr>
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<td></td>
<td>3</td>
<td>0.0489</td>
<td>Kulite</td>
<td>XCL-093-25A</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>4</td>
<td>0.0865</td>
<td>Kulite</td>
<td>XCL-093-25A</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>5</td>
<td>0.1340</td>
<td>Kulite</td>
<td>XCL-152-25A</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>6</td>
<td>0.1910</td>
<td>Kulite</td>
<td>XCL-152-25A</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>7</td>
<td>0.2569</td>
<td>Kulite</td>
<td>XCL-152-25A</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>8</td>
<td>0.3309</td>
<td>Kulite</td>
<td>XCL-152-25A</td>
</tr>
<tr>
<td>9</td>
<td></td>
<td>9</td>
<td>0.4122</td>
<td>Kulite</td>
<td>XCL-152-25A</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>10</td>
<td>0.5000</td>
<td>Kulite</td>
<td>XCL-152-25A</td>
</tr>
<tr>
<td>11</td>
<td></td>
<td>11</td>
<td>0.5933</td>
<td>Kulite</td>
<td>XCL-152-25A</td>
</tr>
<tr>
<td>12</td>
<td></td>
<td>12</td>
<td>0.6910</td>
<td>Endevco</td>
<td>8515C-15</td>
</tr>
<tr>
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<td>0.7921</td>
<td>Endevco</td>
<td>8515C-15</td>
</tr>
<tr>
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<td></td>
<td>14</td>
<td>0.8955</td>
<td>Endevco</td>
<td>8515C-15</td>
</tr>
<tr>
<td>15</td>
<td></td>
<td>15</td>
<td>0.9750</td>
<td>Endevco</td>
<td>8515C-15</td>
</tr>
</tbody>
</table>

2.5.1.1 Kulite transducers

The Kulite models range from 0.062 in. − 0.152 in. in diameter, with the smallest diameter model being implemented in the high curvature area of the leading edge.
Figure 2.7 demonstrates the size scale of the transducers used in the leading edge. The transducers operate by piezoresistive excitation. The pressure-measuring diaphragm is one leg of a wheatstone bridge, changing its resistive value when deflected by pressure fluctuations. The transducers have an absolute pressure resolution of 0 psi−25 psi and a natural frequency of 240 kHz. Each transducer is also temperature compensated in the range $T = [80^\circ F, 180^\circ F]$ and offers an operational temperature range of $T = [-65^\circ F, 250^\circ F]$. All transducers are positioned as close as possible to the surface of the airfoil without causing any hydrodynamic irregularities.

2.5.1.2 Endevco transducers

The Endevco model used (8515C-15) is 0.030 in. thick and 0.250 in. in overall diameter. The thin, surface mount type sensor allows the pressure at the trailing
edge of the model, where the airfoil becomes very thin, to be acquired. Each sensor and wire is recessed into the model, so as to lie flush with the surface. Figure 2.8 shows the size of the transducer and the flush mounted wire leads. Like the Kulite models, the Endevcos are high sensitivity piezoresistive transducers. The transducer model used has an absolute pressure range of 0 psi–15 psi, resulting in a finer resolution than that offered by the on-board Kulites. The natural frequency is 180 kHz, and the sensors have a compensated temperature range of $T = [-65^\circ F, 250^\circ F]$.

Figure 2.8. Size of trailing edge Endevco (8515C-15) and wiring

2.5.2 Freestream pressure

The freestream static and total pressures are measured upstream of the splitter plate assembly via a pitot-static probe. Each output of the probe is connected to a Setra model 270 high accuracy, absolute, barometric pressure transducer. The
Setra model 270 transducers used have an accuracy of ±0.03% FS with a 0.027% FS linearity. The measurable pressure range is 8.70 psi–15.95 psi (600 mbar–1100 mbar). The calibration slopes for each airfoil pressure transducer, as well as the no-flow zeroes, were determined using a Setra model 270, which will be discussed in §2.6.1.

2.5.3 Angle of attack

The instantaneous angle of attack is monitored by connecting the wire shaft to a Positek RIPS P500 inductive rotary sensor, fixed to the tunnel outer window (Figure 2.9). The sensor has a linear output of 0V–5V over the range 0°–60°. The output has a frequency response greater than 10 kHz and a noise level less than 0.02% FSO.

Figure 2.9. Wire-side angle of attack sensor connection
2.6 Calibration

2.6.1 Airfoil surface pressure

The airfoil surface pressure transducers (Kulites and Endevcos) were calibrated at the beginning of each experimental testing session to account for any drifts of supply voltage during non-operational times and, also, any changes of the amplification circuitry setup. The sensitivity slopes of each sensor are determined by applying a least-squares fit to an applied pressure versus output voltage curve. This curve is produced by monitoring the final output voltage of the transducer, after amplification, and the applied pressure at the surface, which is measured using the high accuracy Setra model 270. During the calibration, the applied pressure is varied continuously by pulling a suction pressure on the transducer and slowly releasing it. A typical output is shown in Figure 2.10. The offsets of each transducer are obtained by measuring the atmospheric pressure during a no-flow acquisition before each run set of the test matrix.

2.6.2 Test section calibration

Because of the blockage introduced by the splitter plate assembly (Figure ??), there is a large flow acceleration requiring a calibration between the freestream static pressure measured at $P_1$, the acquisition location during testing, and the static pressure between the splitter plates, measured at $P_2$, also shown in Figure 2.11. The calibration is completed with the airfoil removed from the test section. The upstream and downstream static pressure and total pressure are then measured as fan speed is increased in increments of 5 Hz. The resulting data set is then fit using a third-order polynomial to achieve the calibration equation
Figure 2.10. Dynamic calibration with linear fit

\[
\left( \frac{P_0}{P_S} \right)_2 = 6.6522 \left( \frac{P_0}{P_S} \right)_1^3 - 19.899 \left( \frac{P_0}{P_S} \right)_1^2 + 21.236 \left( \frac{P_0}{P_S} \right)_1 - 6.9898. \quad (2.3)
\]

The downstream total pressure is taken to be the highest pressure measured on the airfoil. It should be noted that no blockage corrections are applied to the dynamic data set, as this is not well defined for unsteady wind tunnel testing.

2.7 Data acquisition

All instantaneous surface pressure, freestream pressure, and angle of attack signals are simultaneously acquired by a Microstar Laboratories DAP5380a and three MSXB 028 simultaneous sampling boards at a sampling frequency of \( f_s = 5 \text{ kHz} \). The surface pressure signals first pass through an instrumentation amplification circuit, controlling the gain and offset of each individual signal, and are subsequently low-
pass, anti-alias filtered at 2.5 kHz before reaching the sample-and-hold boards. The analog-to-digital conversion has a 12-bit resolution due to the MSXB 028 boards. Raw quantization levels of all signals are stored in text files for further reduction and post-processing.

2.8 Aerodynamic load calculation

A Fortran program is used to convert all raw signal values to dimensional form using the appropriate calibrations. Non-dimensional aerodynamic loads are then computed. The pressure coefficient, $C_p$, is determined using a relationship accounting for compressibility effects and defined as

$$C_p = \frac{2}{\gamma M_{\infty}^2} \left( \frac{P_{s,\text{airfoil}}}{P_{s,\infty}} - 1 \right),$$

(2.4)

where $\gamma$ is the specific heat ratio, $P_{s,\text{airfoil}}$ is the surface pressure on the airfoil,
and $P_{s,\infty}$ is the freestream static pressure, determined by applying the test section calibration. In compressible flow regimes the possible existence of supersonic flow and shock development can be determined through the local Mach number. The local Mach number, $M_l$, on the airfoil is calculated as

$$M_l = \sqrt{\frac{2}{\gamma - 1} \left( \frac{P_0}{P_{s,airfoil}} \frac{\gamma - 1}{\gamma} - 1 \right)}, \quad (2.5)$$

where $P_0$ is the freestream total pressure.

![Figure 2.12. Airfoil coordinate systems](image)

The integrated aerodynamic loads are derived from the $C_p$ values and using the airfoil coordinate system described in Figure 2.12. A trapezoidal integration scheme is used to compute the integrals. The non-dimensional normal and axial loads, corresponding to the $N$ and $A$ coordinate system, are given as

$$C_n = \int_0^1 \left( C_p^p - C_p^s \right) d \left( \frac{x}{c} \right), \quad (2.6)$$
\[ C_a = \int_0^1 \left( C_p^P \frac{dy^P}{dx} - C_p^S \frac{dy^S}{dx} \right) d \left( \frac{x}{c} \right) , \]  

(2.7)

where the superscripts \( P \) and \( S \) refer to the pressure (lower) and suction (upper) surfaces of the airfoil, respectively. The lift and drag coefficients are then found through a coordinate rotation transformation

\[ C_l = C_n \cos(\alpha) + C_a \sin(\alpha) , \]  

(2.8)

\[ C_d = C_n \sin(\alpha) - C_a \cos(\alpha) . \]  

(2.9)

The leading edge moment is defined as

\[ C_{mLE} = -\int_0^1 \left( C_p^P - C_p^S \right) \frac{x}{c} d \left( \frac{x}{c} \right) , \]  

(2.10)

which can be converted to the quarter-chord pitching moment using the normal force and moment arm

\[ C_{m_{c/4}} = C_{mLE} + C_n/4 . \]  

(2.11)
CHAPTER 3

MODEL FRAMEWORK AND REVIEW OF ITS METHODS

3.1 Framework

The non-dimensional pressure field around a pitching airfoil can be expressed in the following manner:

\[
C_p(\xi, \alpha, \dot{\alpha}) = C_{p, inv}(\xi, \alpha) + C_{p, visc}(\xi, \alpha) + C_{p, unsteady}(\xi, \alpha, \dot{\alpha}),
\]

(3.1)

where \(C_{p, inv}\) is the inviscid pressure field, \(C_{p, visc}\) is the effect of viscosity on the steady pressure, \(C_{p, unsteady}\) is the pressure field component due to \(\dot{\alpha} \neq 0\), and \(\xi\) is the non-dimensional chord-wise coordinate, \(x/c\). This dissertation will consider two models of the surface pressure field to infer effects associated with airfoil geometry on the dynamic stall phenomenon. Namely, the following two pressure fields will be modeled:

\[
C^*_p = C_{p, unsteady}(\xi, \alpha, \dot{\alpha}) - \epsilon C_{p, inv}(\xi, \alpha),
\]

(3.2)

\[
\dot{C}_p = C_{p, unsteady}(\xi, \alpha, \dot{\alpha}) - C_{p, steady}(\xi, \alpha).
\]

(3.3)

These two systems correspond to viewing the pressure field from different frames of reference. The first, expressed in Eq. 3.2 will be termed the “star” reference frame and includes steady, viscous effects in the unsteady pressure field. Likewise, pressure
fields expressed as in Eq. 3.3 will be denoted the “hat” reference frame and only consider the pressures associated with the unsteady motion.

Within each model two types of airfoil geometries are considered. The first is called the reference airfoil geometry. This airfoil has an arbitrarily chosen geometry and supplies all of the unsteady pressure field data with which to calculate the model parameters. The second airfoil type is called the candidate airfoil and is a geometry that one desires to estimate the unsteady pressure field/ aerodynamic loading for a given pitch trajectory. For this geometry, only the steady pressure field characterization is required.

![Diagram of model schematic for “star” reference frame]

Figure 3.1. Model schematic for “star” reference frame

The components of the low-order dynamic stall model described by the “star” reference frame system are outlined schematically in Figure 3.1. The different com-
Components of the model are colored by the chapters where their respective derivation appears. This low-order model is composed of three essential sections: the calculation of parametric proper orthogonal decomposition (PPOD) modes (or $\psi -$modes) for the known unsteady surface pressures of the reference geometry, the calculation of split modes ($\theta -$modes) for a candidate geometry from its known steady surface pressures, and the approximation of unknown modal coefficients for both the PPOD modes and the split modes for the candidate geometry given a motion trajectory.

Reference

airfoil geometry:

$C_p(x/c, t)$ (known)

Remove steady from unsteady:

$\hat{C}_p = C_p - C_p^{\text{steady}}$

Calculate paramter dependent POD modes of $C_p$: $\psi -$modes

Candidate

airfoil geometry:

$C_p, C_{\theta}(x/c, t)$ (unknown)

Calculate modal coefficient approximations: $d_{i,j}$

Calculate candidate pressure field from given trajectory input: $\alpha(t)$

Calculate PPOD modes from $\phi -$modes: $\hat{\psi} -$modes

Chapter 5

Not in dissertation

Figure 3.2. Model schematic for “hat” reference frame

Components describing the low-order dynamic stall model derived from the “hat” reference frame system are outlined schematically in Figure 3.2. Again, the components of the model are colored by the chapters where they appear. This low-order model only has two primary sections: the calculation of parametric proper orthogonal decomposition (PPOD) modes (or $\psi -$modes) for the known unsteady surface pressures of the reference geometry and the approximation of unknown modal coefficients for the PPOD modes for the candidate geometry given a motion trajectory.
Note that this model does not require the calculation of any geometry-dependent split modes. This subtlety will be discussed in Chapter ??.

3.2 Calculation methods

Because this model relies heavily on multiple proper orthogonal decomposition techniques, it seems appropriate to first review the development of these decompositions and their implementations.

3.2.1 Proper orthogonal decomposition

The application of proper orthogonal decomposition to fluid mechanics was originally proposed by Lumley [32] as a means of extracting coherent flow structures from random turbulent fields. Likewise, it has been used extensively in the analysis of particle imaging velocimetry (PIV) data sets to determine the most fundamental topological features of the given flow fields. In more general terms, the proper orthogonal decomposition is a spectral decomposition of a given function, $u = u(x, t)$, resulting in a set of orthogonal “spatial” modes (associated with the variable $x$) that are optimal when considering the signal energy convergence. This decomposition takes the form of a series representation:

$$u(x, t) = \sum_i a_i(t)\phi_i(x),$$

(3.4)

where $a_i(t)$ are temporal coefficients describing the dynamic behavior of each spatial mode, $\phi_i(x)$ [32]. The resulting empirical modes capture the energy of the function more rapidly than any other basis of the same dimension. The modes are derived by maximizing the function

$$I = \frac{\langle|u, \phi|^2\rangle}{||\phi||^2},$$

(3.5)
where $(\cdot, \cdot)$ is an inner product for the $L^2(\Omega_x)$ space of square-integrable functions and $|| \cdot ||$ is the respective norm. The inner product is defined as

$$(f, g) = \int_{\Omega_x} (f \cdot g) dx,$$  

resulting in a squared norm of the form

$$||\phi||^2 = (\phi, \phi) = \int_{\Omega_x} |\phi|^2 dx,$$  

where $\Omega_x$ is the spatial domain over which $u$ and $\phi$ are defined. In Eq. 3.5, $\langle \cdot \rangle$ is an averaging operator over the time domain. Using variational calculus and the definitions in Eq. 3.6 and Eq. 3.7, the optimization problem of Eq. 3.5 is cast into an integral eigenvalue problem

$$\int_{\Omega_x} \langle u(x) \cdot u(x') \rangle \phi(x') dx' = \lambda \phi(x),$$  

where $\langle u(x) \cdot u(x') \rangle = R(x, x')$ is the temporally averaged autocorrelation function of $u(x, t)$ and $\lambda$ is the corresponding eigenvalue [25]. From this result, the spatial modes are the eigenvectors of Eq. 3.8 and are prioritized according to their relative energy by comparing their respective eigenvalues: relative energy $\equiv \lambda_i / \sum_j \lambda_j$. Once the eigenvalue problem is solved, the temporal coefficients, $a_i(t)$, are found by projecting the function, $u(x, t)$, onto the spatial modes, $\phi_i(x)$, i.e.

$$a_i(t) = \int_{\Omega_x} (u(x, t) \cdot \phi_i(x)) dx.$$  

3.2.2 POD calculation on a discrete domain

Because POD is often applied to discrete data rather than continuous functions, the integrals of the inner product and squared norm of Eq. 3.6 and Eq. 3.7, respectively, along with the time averaging operator must be discretized. To begin, the
order of operation can be switched between the averaging and spatial integration because the first function in the average, \( u(x) \), is not in the integrated domain:

\[
\frac{1}{N_T} \sum_{k=1}^{N_T} u(k)(x) \int_{\Omega} u(k)(x') \phi(x') dx' = \lambda \phi(x),
\]

(3.10)

where \( N_T \) is the total number of time steps. The spatial integrals are then approximated using a trapezoidal quadrature method, i.e.

\[
\int_{\Omega_x} (f \cdot g) dx \approx \sum_{i=1}^{n_x} \omega_i f(x_i) g(x_i),
\]

(3.11)

where \( n_x \) is the total number of discrete spatial points and \( \omega \) is a weighting function defined as

\[
\omega = \Delta x [0.5 \ 1 \ \cdots \ 1 \ 0.5]
\]

(3.12)

on an equally spaced domain. Here, \( \Delta x \) is the spatial separation between data points. For a non-uniform spacing, the weighting function becomes

\[
\omega = \begin{bmatrix}
\Delta x_1/2 & (\Delta x_1 + \Delta x_2)/2 & \cdots & (\Delta x_{i-1} + \Delta x_i)/2 & \cdots & (\Delta x_{n_x-1} + \Delta x_n)/2
\end{bmatrix}
\]

\[
\omega_{n_x}
\]

(3.13)

where \( \Delta x_i = x_{i+1} - x_i \). Applying this integration routine to the POD problem, the left side of Eq. 3.10 becomes

\[
\frac{1}{N_T} \sum_{k=1}^{N_T} \sum_{i=1}^{n_x} \left[ \sqrt{\omega_i} u(k)(x_i) \sqrt{\omega_i} \phi(x_i) \right] = \lambda \phi(x_j),
\]

(3.14)

where the weights have been split to define the hat terms, ( \( \hat{\cdot} \) ). Condensing the spatial integration to matrix form yields
\[
\frac{1}{N_T} \sum_{k=1}^{N_T} u(k)(x_j) \hat{u}(k) \hat{\phi} = \lambda \phi(x_j), \quad \text{for } j = 1, \ldots, n_x. \quad (3.15)
\]

Now, both sides are multiplied by $\sqrt{\omega_j}$ for $j = 1, \ldots, n_x$ to give

\[
\frac{1}{N_T} \sum_{k=1}^{N_T} \hat{u}(k) \hat{u}(k) \hat{\phi} \equiv \tilde{A} \hat{\phi} = \lambda \hat{\phi}. \quad (3.16)
\]

Here, $\tilde{A}$ is a $n_x \times n_x$, Hermitian operator for the eigenvalue problem defined as

\[
\tilde{A}_{i,j} = \frac{1}{N_T} \sum_{k=1}^{N_T} \left( \sqrt{\omega_j} u(k)(x_j) \right) \left( \sqrt{\omega_i} u(k)(x_i) \right). \quad (3.17)
\]

The POD spatial modes are then recovered by “unweighting” the eigenvectors from Eq. 3.16 i.e.

\[
\phi_{POD}(x_i) = \frac{\hat{\phi}(x_i)}{\sqrt{\omega_i}}. \quad (3.18)
\]

Likewise, the projected coefficients of the reconstruction are determined using the trapezoidal quadrature as

\[
a_i(t) = \int_{\Omega_x} (u \cdot \phi_i) \, dx \approx \sum_{j=0}^{n_x} \omega_j u(x_j, t) \phi_j(x_j), \quad (3.19)
\]

where $\phi(x) = \phi_{POD}$ is the unweighted POD spatial mode from Eq. 3.18.

### 3.2.3 Parametric POD

Motivated by active flow control, Gordeyev and Thomas [14] recently introduced a new POD technique in order to capture the dynamics of a flow field having two distinct characteristic flow states. This technique determines optimal modes for each flow state as well as the transients existing from the trajectories between the two states. While this technique was applied to study active bluff body flow control, it also
offers a means of determining characteristic modes of multi-parameter functions, such as the flow field events associated with dynamic stall. Considering then a function of multiple parameters, such as the surface pressure field of an airfoil during a dynamic stall event, the eigenvalue problem from Eq. 3.8 becomes

$$\int R(\xi, \xi'; k, \alpha_0, \alpha_1) \phi_i(\xi'; k, \alpha_0, \alpha_1) d\xi' = \lambda_i(k, \alpha_0, \alpha_1) \phi_i(\xi; k, \alpha_0, \alpha_1), \quad \|\phi_i\|^2 = 1,$$

(3.20)

where $R(\xi, \xi'; k, \alpha_0, \alpha_1)$ is the temporally averaged autocorrelation function of the surface pressure field, $p(\xi, t; k, \alpha_0, \alpha_1)$. The eigenvectors and eigenvalues are now contained within subsets of reduced frequency $k$, mean angle of attack $\alpha_0$, and pitch oscillation amplitude $\alpha_1$. Here, the spatial coordinate is the non-dimensional chordwise location, $\xi = x/c$. The modes are also restricted to be orthonormal. The pressure field is then reconstructed in the low-dimensional space spanned by the eigenvectors as in Eq. 3.4

$$p(\xi, \alpha(t), \dot{\alpha}(t); k, \alpha_0, \alpha_1) = \sum_i a_i(\alpha(t), \dot{\alpha}(t); k, \alpha_0, \alpha_1) \phi_i(\xi; k, \alpha_0, \alpha_1),$$

(3.21)

where the parameters $\alpha(t)$ and $\dot{\alpha}(t)$ are the angle of attack and pitch rate, respectively, and describe the state of motion of the airfoil. These values can be parameterized in time, $t$, as shown, however, for the remainder of this dissertation they will be shown simply as $\alpha$ and $\dot{\alpha}$.

To further extract common features of the parameter-based dataset, specifically the fundamental modes describing all subsets, it is desirable to obtain a set of purely spatial modes by subsequently applying a POD expansion to the $\phi$—modes as
\[ \phi_i(\xi; k, \alpha_0, \alpha_1) = \sum_j b_{i,j}(k, \alpha_0, \alpha_1) \psi_{i,j}(\xi), \quad ||\psi_{i,j}||^2 = 1. \] (3.22)

The resulting \( \psi \)–modes are the optimal set of purely spatial modes describing the entire parameter space considered. This approach is termed parametric POD (PPOD). The parametric form of the decomposition maintains a similar form to Eq. 3.4,

\[ p(\xi, \alpha; k, \alpha_0, \alpha_1) = \sum_{i,j} d_{i,j}(\alpha; k, \alpha_0, \alpha_1) \psi_{i,j}(\xi), \quad ||\psi||^2 = 1, \] (3.23)

where the modal coefficients, \( d_{i,j} \), are determined, as in Eq. 3.9, by projecting directly onto \( \psi_{i,j}(\xi) \).

### 3.2.4 Split mode POD

Split mode POD was derived by Camphouse et al.\(^3\) for flow control applications to, again, model a system that possesses two distinct flow states. The basic idea of this technique is to derive a basis for a given flow state and then determine the components of a secondary flow state that are orthogonal to this basis. This normal set is then combined with the original basis to create a low-order representation of both flow states. The normal set describes new flow dynamics that are introduced by the secondary flow state. While Camphouse considered flow states created by the application of flow control (\textit{i.e.} control off and control on states), this work will apply split mode POD to account for a change in airfoil geometry. That is, it will be used to account for the difference in geometry between the reference and candidate airfoils.

In general, the split mode POD set, termed \( \theta \)–modes, is calculated in the following manner. Consider two spatio-temporal flow states described by \( u_1(x, t) \) and \( u_2(x, t) \). POD is applied to the first state, yielding spatial modes, \textit{i.e.}
\[ u_1(x, t) = \sum_i a_i(t)\phi_i(x). \quad (3.24) \]

The second flow state is then projected onto the basis describing the first state,

\[ b_i(t) = \int u_2(x, t)\phi_i(x)dx. \quad (3.25) \]

The reconstruction of the second state is removed from the actual data,

\[ \tilde{u}_2(x, t) = u_2(x, t) - \sum_i b_i(t)\phi_i(x), \quad (3.26) \]

where \( N_1 \) is the number of modes required to adequately reconstruct the first state. POD is then applied to \( \tilde{u}_2(x, t) \) to produce a set of modes that are normal to \( \phi(x) \). That is,

\[ \tilde{u}_2(x, t) = \sum_j g_j(t)\theta_j(x), \quad (3.27) \]

where \( g(t) \) and \( \theta(x) \) are the split mode dynamic coefficients and mode shapes, respectively. As before, \( N_2 \) is the number of modes required to adequately reconstruct \( \tilde{u}_2(x, t) \). The final low-order representation of the secondary flow state is then

\[ u_2(x, t) = \sum_i b_i(t)\phi_i(x) + \sum_j g_j(t)\theta_j(x). \quad (3.28) \]

The first summation represents all the flow dynamics that are similar to those found in the first flow state, while the second summation describes the new flow dynamics that differentiate the secondary flow state from the first.
CHAPTER 4

STAR METHOD PPOD CALCULATION

The first focus in developing the low-order dynamic stall model is to define the mode shapes, or basis, of the system. This chapter demonstrates the calculation of this basis set for the “star” reference frame, which is defined as the parametric POD modes of the unsteady, surface pressure field for an arbitrary reference airfoil geometry.

4.1 “Star” reference frame

To discern the dynamics of the surface pressure field associated with the viscous, unsteady flow over the pitching airfoil, a change of reference frame is considered. Namely, the steady, inviscid pressure field is removed from the experimental dataset. The steady, inviscid surface pressure is provided via a Smith-Hess panel method [9] written in MATLAB. Figure 4.1 shows a comparison of the non-dimensional chordwise surface pressure, $C_{p,steady}$, of the NACA 23012 acquired experimentally and the steady, inviscid panel method pressure, $C_{p,inv}$. The case shown represents a static ($\dot{\alpha} = 0$), attached flow at $\alpha = 9.8^\circ$. This comparison shows that the two pressure fields follow similar trends with $\xi$; however, quantitatively, the experimental pressure field is bounded by the panel method solution, resulting in a decreased integrated normal force slope, $\partial C_n/\partial \alpha$, of the former. Figure 4.2 demonstrates this point more completely where $\partial C_n/\partial \alpha$ for the experimental data is shown to be lower than that of the computed inviscid values. In order to assess the generality of the static experimental results, the $C_n$ values from two other experiments involving a NACA 23012
produced by NACA[26] and Leishman[28] are also displayed. The Reynolds number of the current experiment is \( \text{Re}_c = 1.12 \times 10^6 \), and for the NACA and Leishman experiments \( \text{Re}_c = 1.6 \times 10^6 \). Here, excellent agreement is realized in \( \partial C_n / \partial \alpha \) for all three experiments, and the stall angle of the current dataset matches that of Leishman. The difference in the maximum value of \( C_n \) of the current dataset and Leishman’s data likely exists due to the fact that no blockage corrections are applied to the current values. The discrepancy between the current experimental data and the results of the panel method are, therefore, attributed to Reynolds number (\( i.e. \) viscous effects) where the latter represents the case of \( \text{Re}_c \to \infty \). To account for this effect on a specified airfoil geometry operating at a given Reynolds number, a scaling term is introduced in the change of reference frame defined as

\[
\epsilon = \frac{\left. \frac{dC_n}{d\alpha} \right|_{\text{steady}}}{\left. \frac{dC_n}{d\alpha} \right|_{\text{inv}}}, \quad (4.1)
\]

where the subscript \( \text{steady} \) denotes the \( C_n \) slope corresponding to the full steady
pressure field term described in Eq. 3.1. This scaling parameter attempts to impose pseudo-viscous effects on the inviscid pressure field without allowing for any type of stall occurrence. In this way, a post-stall, steady pressure field is realized by which the unsteady pressure field can be compared. The appropriately scaled panel method solution is also shown in Figure 4.2.

![Figure 4.2. Comparison of NACA 23012 static $C_n$](image)

The scaled inviscid (or pseudo-viscous) pressure field is then removed from the experimentally acquired unsteady pressure fields, leaving the unsteady surface pressures defined as

$$C_p^*(\xi, \alpha, \dot{\alpha}) = C_{p,\text{unsteady}}(\xi, \alpha, \dot{\alpha}) - \epsilon C_{p,\text{inv}}(\xi, \alpha).$$

(4.2)

By moving to this reference frame, the PPOD analysis produces spatial modes of
the $C_p^*$ function that represent, by definition, perturbations from the pseudo-viscous, attached flow steady pressure field.

This process of moving reference frames has been demonstrated using the NACA 23012 airfoil due to the body of literature that supports the current experimental results, which is not available for the proprietary airfoils. However, it should be noted that moving to the “star” reference frame is a general procedure that must be calculated for any arbitrarily chosen reference airfoil.

4.2 $\phi$-mode calculation

Using the decomposition described in Eq. 3.4 each independent parameter case in the dataset of $C_p^*(\xi, \alpha, \dot{\alpha}; k, \alpha_0)$ for the reference airfoil geometry is decomposed into $\phi$-spatial modes. That is to say, $\phi = \phi(\xi; k, \alpha_0)$. The reference geometry for this analysis is arbitrarily taken to be the PA2 airfoil. Each independent parameter case considered for the PA2 is listed in Table 4.1 where entries move from unsteady attached flow (AF), through light stall (LS) to deep stall (DS) with increasing case number. Thus, the database used for computation of the POD basis encompasses all of the anticipated flow regimes.

Figure 4.3 shows the eigenvalue magnitudes — representing the modal energy — corresponding to the $i$th $\phi$-mode of each decomposition. Here, each case is sorted by reduced frequency, $k$, and drawn as a function of mean angle of attack, $\alpha_0$. For clarity, only the first five eigenvalues and the sum total of all the eigenvalues are shown. Comparison of the mode 1 eigenvalues with the sum $\sum \lambda_i$ indicate that essentially all of the $C_p^*$ fluctuation amplitude is carried by $\phi_1(\xi; \alpha_0, k)$. The eigenvalues also suggest a trend of increasing energy with increasing mean angle for each mode, which is to be expected as the development of leading edge vortices coincides with higher $\alpha_0$ as well as larger deviations from the pseudo-viscous steady attached flow behavior.
TABLE 4.1

PA2 PARAMETER CASES FOR “STAR” METHOD ($\alpha_1 = 8^\circ$ AND $M = 0.2$ FOR ALL CASES)

<table>
<thead>
<tr>
<th>Case #</th>
<th>$\alpha_0$</th>
<th>$k$</th>
<th>Type</th>
<th>Case #</th>
<th>$\alpha_0$</th>
<th>$k$</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7°</td>
<td>0.020</td>
<td>AF</td>
<td>16</td>
<td>12°</td>
<td>0.020</td>
<td>DS</td>
</tr>
<tr>
<td>2</td>
<td>7°</td>
<td>0.050</td>
<td>AF</td>
<td>17</td>
<td>12°</td>
<td>0.050</td>
<td>DS</td>
</tr>
<tr>
<td>3</td>
<td>7°</td>
<td>0.0757</td>
<td>AF</td>
<td>18</td>
<td>12°</td>
<td>0.0757</td>
<td>DS</td>
</tr>
<tr>
<td>4</td>
<td>8°</td>
<td>0.020</td>
<td>LS</td>
<td>19</td>
<td>13°</td>
<td>0.020</td>
<td>DS</td>
</tr>
<tr>
<td>5</td>
<td>8°</td>
<td>0.050</td>
<td>LS</td>
<td>20</td>
<td>13°</td>
<td>0.050</td>
<td>DS</td>
</tr>
<tr>
<td>6</td>
<td>8°</td>
<td>0.0757</td>
<td>LS</td>
<td>21</td>
<td>13°</td>
<td>0.0757</td>
<td>DS</td>
</tr>
<tr>
<td>7</td>
<td>9°</td>
<td>0.020</td>
<td>LS</td>
<td>22</td>
<td>14°</td>
<td>0.020</td>
<td>DS</td>
</tr>
<tr>
<td>8</td>
<td>9°</td>
<td>0.050</td>
<td>LS</td>
<td>23</td>
<td>14°</td>
<td>0.050</td>
<td>DS</td>
</tr>
<tr>
<td>9</td>
<td>9°</td>
<td>0.0757</td>
<td>LS</td>
<td>24</td>
<td>14°</td>
<td>0.0757</td>
<td>DS</td>
</tr>
<tr>
<td>10</td>
<td>10°</td>
<td>0.020</td>
<td>LS</td>
<td>25</td>
<td>15°</td>
<td>0.020</td>
<td>DS</td>
</tr>
<tr>
<td>11</td>
<td>10°</td>
<td>0.050</td>
<td>LS</td>
<td>26</td>
<td>15°</td>
<td>0.050</td>
<td>DS</td>
</tr>
<tr>
<td>12</td>
<td>10°</td>
<td>0.0757</td>
<td>LS</td>
<td>27</td>
<td>15°</td>
<td>0.0757</td>
<td>DS</td>
</tr>
<tr>
<td>13</td>
<td>11°</td>
<td>0.020</td>
<td>LS</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>11°</td>
<td>0.050</td>
<td>LS</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>11°</td>
<td>0.0757</td>
<td>LS</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 4.4 shows the first five $\phi$-modes for attached flow pitching (case #1), light stall (case #14), and deep stall (case #24). Here, negative and positive spatial coordinates refer to the bottom (pressure) side and the top (suction) side of the airfoil, respectively. The leading edge coordinate is located at $\xi = 0$. The corresponding modal coefficients, $a_i(\alpha; \alpha_0, k)$, are given in Figure 4.5 where each line represents the projection of the phase averaged $-C_p^*$ onto that respective mode. The abscissa is given as the stall penetration angle, $\alpha_{sp} = \alpha - \alpha_{ss}$. The magnitudes of the modal coefficients demonstrate the dominance of mode 1, which is an order of magnitude greater than the higher modes.
4.3 φ-mode reconstruction

To evaluate the accuracy of the φ-mode reconstruction, a comparison with the integrated aerodynamic loads is considered. Similar to the surface pressure field, the unsteady aerodynamic loads used for comparison are defined as deviations from the pseudo-viscous steady attached flow behavior. In particular,

\[ C_n^* = C_{n,exp} - \epsilon C_{n,panel}, \]
\[ C_m^* = C_{m,exp} - \epsilon C_{m,panel}. \]  

The non-dimensional load differences being compared are the normal force, \( C_n^* \), and the quarter-chord pitching moment, \( C_m^* \), because of their importance to the rotor dynamicist when developing airfoil sections. The equivalent aerodynamic loading values for the reconstruction are determined as
\( C_{n,i}^{\star, POD} = a_i \int_0^1 \left( \phi_i^+ (\xi) - \phi_i^- (\xi) \right) d\xi, \)  

\( C_{m,i}^{\star, POD} = -a_i \int_0^1 \left( \phi_i^+ (\xi) - \phi_i^- (\xi) \right) \xi d\xi + C_{n,i}^{\star, POD} / 4, \)  

where the superscript on \( \phi \) represents the suction side (+) or pressure side (−) of the airfoil.
To quantify the error of the POD approximation with respect to the experimental data, the following definition is used:

\[
\text{Error} = \left( \frac{1}{N_s} \sum \left( \langle C_x^* \rangle - \left\langle \sum_{i=1}^{N_{mode}} C_{x,i}^{POD} \right\rangle \right)^2 \right)^{1/2}, \quad (4.7)
\]

where \( N_s \) is the number of samples being considered (time stamps), \( \langle \cdot \rangle \) is the phase averaging operator, \( N_{mode} \) is the current number of modes used for the approximation, and the subscript \( x \) is a dummy variable for the aerodynamic load being considered.

Figure 4.5. PA2 \( \phi \)-mode coefficients, \( a_i \), for attached flow pitching, light stall, and deep stall. Note change in ordinate scale after the first mode.
Note that this is an absolute error.

The \( \phi \)-mode approximations using up to five modes are shown in Figure 4.6 and Figure 4.7 for \( C_n^* \) and \( C_m^* \), respectively. The ordinate label indicates the number of terms included in the summation. Phase averaged values of the experimental data are shown in black with error bars indicating the standard deviation. The \( \phi \)-mode reconstructions are drawn in color. The respective absolute error magnitudes are shown in Figure 4.8. Notice that the ordinate scale of the normal force figure is an order of magnitude larger than that of the quarter-chord moment. That being said,
the normal force converges to error values on the order of the moment error with the addition of the second mode, demonstrating the rapid error convergence of the normal force load. The most interesting behavior of the error values, seen in both loads, is that the values for each pitching case appear to converge at the addition of mode 5. This mode is pivotal in that large error reductions are seen before it while the subsequent addition of higher modes results in little error change, which is due primarily to the fact that there is not much room for further improvement because the error has already been significantly reduced. Comparing the error magnitudes

Figure 4.7. PA2 $\phi$-mode quarter-chord moment, $C_m^*$, reconstruction for attached flow pitching, light stall, and deep stall
for a five-mode reconstruction of each load to the respective amplitude range of the experimental deep stall case, being 1.25 for the normal force and 0.32 for the quarter-chord moment, results in the following error percentages:

\( C_n^{\text{five-mode reconstruction}} \)
- Attached flow pitching: 0.7%,
- Light stall: 0.4%,
- Deep stall: 0.9%,

\( C_m^{\text{five-mode reconstruction}} \)
- Attached flow pitching: 0.7%,
- Light stall: 0.7%,
- Deep stall: 1.3%.

With this level of quantitative error convergence and the qualitative behavior of the reconstructions exhibited in Figure 4.6 and Figure 4.7, it is clear that a five-mode reconstruction at each parameter point, \((\alpha_0, k)\), is more than adequate for capturing the dynamics associated with the time evolution of the unsteady surface pressure field of the reference PA2 airfoil.

4.4 Moving from \(\phi\)-modes to \(\psi\)-modes (PPOD)

The decomposition of the previous section provides mode shapes that are optimal at a single point in the parameter space, \(i.e.\) at a given \((\alpha_0, k)\) location. However, it is desirable to obtain a single set of spatial modes that will act optimally over the entire parameter space, meaning for all \((\alpha_0, k)\) contained in the space. This globally optimal set, termed here as \(\psi\)-modes, is obtained by application of parametric POD. These newly defined modes are only functions of the spatial coordinate, \(\xi\), that is \(\psi = \psi(\xi)\); and the corresponding modal coefficients, \(b(\alpha_0, k)\), control the modal magnitude across the parameter space.
The decomposition has the form

\[ \phi_i(\xi; \alpha_0, k) = \sum_j b_{i,j}(\alpha_0, k)\psi_{i,j}(\xi), \]  

(4.8)

where \( \psi_{i,j}(\xi) \) is the \( j \)th modal shape of the \( i \)th \( \phi \)-mode deconstruction. To recover the original pressure signal, \( C^*_{p} \), the reconstruction is

\[ C^*_{p}(\xi, \alpha, \dot{\alpha}; \alpha_0, k) = \sum_i a_i(\alpha, \dot{\alpha}; \alpha_0, k) \sum_j b_{i,j}(\alpha_0, k)\psi_{i,j}(\xi), \]  

(4.9)

where \( a_i(\alpha, \dot{\alpha}; \alpha_0, k) \) are the \( \phi \)-mode coefficients from the previous section. If the
coefficients describing the evolution of each $\phi$-mode to the $\psi$-mode space, $b_{i,j}$, are not required, a contracted version is obtained by explicitly projecting the pressure field onto the $\psi$-modes. This gives

\[ C_p^*(\xi, \alpha, \dot{\alpha}; \alpha_0, k) = \sum_i d_i(\alpha, \dot{\alpha}; \alpha_0, k)\psi_i(\xi), \quad (4.10) \]

where $d_i(\alpha, \dot{\alpha}; \alpha_0, k) = \int C_p^*(\xi, \alpha, \dot{\alpha}; \alpha_0, k)\psi_i(\xi)d\xi$. Here, the $\psi$-modes can be calculated directly from $C_p^*$ by applying POD as in Eq. 3.8. The $\psi$-mode shapes, calculated using the entire parameter space, and the corresponding $d_i$ modal coefficients are shown in Figure 4.9. The modal coefficients are drawn for the same parameter cases studied in the previous section i.e. attached flow ($\alpha_0 = 6.7^\circ, k = 0.020$), light stall ($\alpha_0 = 10.7^\circ, k = 0.051$), and deep stall ($\alpha_0 = 13.9^\circ, k = 0.078$).

4.5 $\psi$-mode reconstruction

The aerodynamic load reconstruction used in §4.3 is again utilized here to evaluate the $\psi$-mode decomposition. Figure 4.10 shows the error, as calculated in Eq. 4.7, of the normal force load and moment load, respectively. While neither load reconstruction converges quite as aggressively as the $\phi$-mode reconstructions, similar trends are observed. The normal force error tends to converge for the different pitching cases at the addition of mode 5 as was seen in the $\phi$-mode reconstructions. While the quarter-chord moment reconstruction appears to continue to taper off in error after mode 5, the error values at this mode are still nearly converged across the pitching cases and on the same order of magnitude as their $\phi$-mode counterparts. Comparing the error magnitudes for a five-mode reconstruction of each load to the respective amplitude of the experimental deep stall case, again being 1.25 for the normal force and 0.32 for the quarter-chord moment, results in the following error percentages:
Figure 4.9. PA2 ψ-mode shapes and coefficients: attached flow pitching (___), light stall (—), and deep stall (—). Note that the ordinate scale of $d_1$ is nearly an order of magnitude larger.

$C_n$ five-mode reconstruction
- Attached flow pitching: 0.8%,
- Light stall: 0.6%,
- Deep stall: 1.2%,

$C_m$ five-mode reconstruction
- Attached flow pitching: 1.9%,
- Light stall: 1.6%,
- Deep stall: 2.0%.
The resulting quarter-chord moment reconstruction error percentages are two times greater than the values seen for the $\phi$-mode reconstructions. Slight increases like this are to be expected because the $\psi$-mode shapes being used are globally optimal over the parameter space, whereas, the $\phi$-mode shapes are locally optimal. Although these values are higher, they are still on the same order of magnitude as the $\phi$-mode reconstructions.

Figure 4.10. PA2 $\psi$-mode aerodynamic load reconstruction error convergence for attached flow pitching (—), light stall (—), and deep stall (—).
Qualitative reconstructions for $C_n^*$ and $C_m^*$ are shown, respectively, in Figure 4.11 and Figure 4.12 for the $\psi$-mode reconstructions. Now, the ordinate label tells the number of terms in the summation and the $\psi$-modes used for the modal aerodynamic load calculation. The flow cases considered are the same as the $\phi$-mode cases, namely, attached flow pitching, light stall, and deep stall. These reconstructions support the results seen in the quantitative error analysis, showing a rapid convergence to the experimental values across the entire pitching trajectory with as few as five modes. The normal force arguably converges for all pitching trajectory cases with as few
as three modes. Likewise, the quarter-chord moment exhibits the correct loading shapes at three modes, but it is not until the addition of the fifth mode that the values fit nicely within the standard deviation bars of the light stall and deep stall cases. This behavior suggests that five modes are, again, adequate for capturing the dynamics associated with the time evolution of the unsteady surface pressure field of the reference PA2 airfoil. More importantly, these five modes are parameter-independent, demonstrating the power of the PPOD technique to obtain optimal spatial modes over the given parameter space. Figure 4.13 shows the deep stall case.
aerodynamic load reconstructions with the inviscid components added to demonstrate the quality of the approximations on the actual air loads.

Figure 4.13. PA2 $\psi$-mode reconstruction of actual $C_n$ and $C_m$ for a deep stall case: $\alpha_0 = 13.9^\circ$, $k = 0.078$
4.6 Projection onto the reference $\psi$-mode basis

The next step in the low-order, “star” reference frame model development is to project the unsteady pressure field of a candidate airfoil onto the reference basis and consider the resulting reconstruction. Using the $\psi$-mode (reference) basis determined in the previous chapter, the projection of the surface pressure field from a candidate geometry is calculated to resolve the respective modal response as

$$d_{CT}^i(\alpha, \dot{\alpha}; \alpha_0, k) = \int C^*_p(\xi, \alpha, \dot{\alpha}; \alpha_0, k) \psi_{REF}^i(\xi)d\xi,$$  \hspace{1cm} (4.11)

where the subscript $CT$ denotes the candidate geometry and, similarly, the subscript $REF$ represents the reference geometry. Ideally, when considering the low-order approximation, the resulting candidate modal coefficients, $d_{CT}^i$, will behave in a similar manner as those of the reference geometry. The reference basis and modal coefficients along with the projections from the PA1 geometry and the NACA 23012 are shown in Figure 4.14. Here, the PA1 and NACA 23012 serve as sample candidate airfoils. The mode shapes are, again, given as a function of non-dimensional chord wise coordinate, $\xi$. Similarly, the modal coefficients are given as functions of stall penetration angle of attack, $\alpha_{sp}$, which is a normalizing factor when comparing geometries due to the fact that $\alpha_{ss}$ is a unique, airfoil-specific parameter. The cases for the PA2 geometry (left coefficients column) are the same as those used in the previous chapter, that is, attached flow pitching ($\alpha_0 = 6.7^\circ$, $k = 0.020$), light stall ($\alpha_0 = 10.7^\circ$, $k = 0.051$), and deep stall ($\alpha_0 = 13.9^\circ$, $k = 0.078$). For purposes of comparison, similar cases are chosen for the PA1 geometry and the NACA 23012. The oscillation amplitude for all the cases of the PA2 and PA1 is $\alpha_1 = 8^\circ$, while it is set at $\alpha_1 = 7.5^\circ$ for the NACA 23012 cases.

While the task of modal coefficient approximation will not be fully considered in this dissertation, a few comments about the behaviors of each contribute mode will be...
Figure 4.14. Candidate airfoil projections onto PA2 $\psi$-modes

PA2: $\alpha_0 = 6.7^\circ$, $k = 0.020$ ( ); $\alpha_0 = 10.7^\circ$, $k = 0.051$ ( ); $\alpha_0 = 13.9^\circ$, $k = 0.078$ ( )

PA1: $\alpha_0 = 6.8^\circ$, $k = 0.020$ ( ); $\alpha_0 = 10.5^\circ$, $k = 0.051$ ( ); $\alpha_0 = 13.4^\circ$, $k = 0.078$ ( )

NACA23012: $\alpha_0 = 7.67^\circ$, $k = 0.051$ ( ); $\alpha_0 = 10.7^\circ$, $k = 0.077$ ( ); $\alpha_0 = 15.7^\circ$, $k = 0.104$ ( )

discussed briefly. The coefficients for each mode show a geometric dependence across the different airfoils but similar trends are still observed for some of the modes. Mode 1 shows a similar response for each airfoil, where larger magnitudes are obtained with
increasing angle of attack. Hysteresis loops are also realized and have two distinct branches no matter what the mean angle of attack describing an ascending path and a descending path. Likewise, mode 2 develops hysteresis loops and larger amplitudes with increasing $\alpha_0$. Again, there appears to be an ascending path and descending path for all cases, although not as well defined as in the mode 1 coefficients. Mode 3 shows a collapse of the coefficients below the static stall angle of attack, $\alpha_{sp} = 0^\circ$, and the development of hysteresis loops beyond this value. The final two modes considered, mode 4 and mode 5, appear to be fine adjustment modes, where the magnitudes are consistently small with the exception of the PA1 mode 5 projections.

4.7 Candidate geometry $\psi$-mode reconstruction

For the remainder of this chapter only the PA1 results will be shown. The corresponding results for the NACA 23012 are given in Appendix B. Following the same methodology of the previous chapter, the modal aerodynamic load reconstructions are considered as a metric for the accuracy of the candidate pressure field projections onto the reference basis. Figure 4.15 and Figure 4.16 show the reconstruction of $C_n^*$ and $C_m^*$, respectively, for the PA1 geometry using the reference $\psi$-modes and the projected coefficients, $d_i^{\psi CT}$, from Eq. 4.11. The same cases for the PA1 geometry as in the previous section are used here to show the reconstruction across the range of pitching motion flow states i.e. attached flow pitching, light stall, and deep stall. The figures are shown as the incremental additions of the respective $\psi$-mode method loading. The ordinate label describes the current summation and the $\psi$-mode and corresponding projection coefficients used to calculate the aerodynamic load. Once again, the abscissa is the stall penetration angle of attack, $\alpha_{sp}$. The experimental aerodynamic loads are shown in black with error bars representing the local standard deviation, and the modal reconstructions are shown in color.

Qualitatively, the normal force reconstruction appears to converge to the exper-
experimental values for the light stall and attached flow pitching cases. It is not until the higher angle of attack values of the 5 mode reconstruction during deep stall that significant deviations from the experimental data are observed. That being said, the 5 mode reconstructions of the quarter-chord pitching moment are not well approximated for the light stall or the deep stall cases. For both cases, the error severely increases with the addition of the last mode, essentially overcorrecting the portions of the pitch cycle at higher angle of attack. This effect causes the magnitude of the light and deep stall moment peaks to be under predicted.
Figure 4.16. PA1 quarter-chord moment reconstruction, $C^*_m$, from projected $\psi$-mode: attached flow pitching, light stall, and deep stall

The quantitative measure of each reconstruction is calculated in a similar fashion to that in §4.3, resulting in an absolute error value for the approximations. These error values are given in Figure 4.17 which includes the errors for attached flow pitching (AF), light stall (LS), and deep stall (DS) for both $C^*_n$ and $C^*_m$. Both loads show the same trends that were observed in the reconstructions, namely, a convergence to the experimental data until mode 5 is added. Ideally, only the first three or four modes would be used for the candidate airfoil reconstruction, however, the exhibited behavior of the fifth mode would not be known a priori. For this
Figure 4.17. PA1 normal force and quarter-chord moment reconstruction error for attached flow pitching (---), light stall (---), and deep stall (---).

reason, a correction modal basis must be determined for the candidate airfoil by the implementation of split modes.

4.8 Candidate geometry split mode calculation

The split mode derivation for flow control applications of Camphouse et al[3] is generalized here to consider flow deviations caused by any parameter. The previous sections of this chapter dealt with resolving the components of the PA1 unsteady surface pressure field that lie in the plane of the PA2 $\psi$-mode basis. Now, the idea is to describe what remains as an additional out-of-plane component, or put simply, the parts of the PA1 pressure field that are orthogonal to the PA2 pressure field. This
new orthogonal set, which is denoted as $\theta$, describes new dynamics of the unsteady surface pressure that are introduced by the geometry change between reference and candidate airfoils. This $\theta$-mode set is calculated as described in §3.2.4. That is, the low-order, $\psi$-mode reconstruction of the PA1 pressure field is, first, removed from the original pressure field

$$
\tilde{C}_p^*(\xi, \alpha, \dot{\alpha}; \alpha_0, k) = C_p^*(\xi, \alpha, \dot{\alpha}; \alpha_0, k) = N_{REF} \sum_i d_i^{CT}(\alpha, \dot{\alpha}; \alpha_0, k) \psi_i^{REF}(\xi), \tag{4.12}
$$

where $N_{REF}$ is the number of reference geometry modes used to adequately resolve the reference pressure field, as determined in the previous chapter. Once this operation is completed, the $\theta$-mode set is calculated as the PPOD of $\tilde{C}_p^*$, or

$$
\tilde{C}_p^*(\xi, \alpha, \dot{\alpha}; \alpha_0, k) = N_{CT} \sum_m g_m(\alpha, \dot{\alpha}; \alpha_0, k) \theta_m(\xi), \tag{4.13}
$$

where $N_{CT}$ is the number of $\theta$-modes required for reconstruction of $\tilde{C}_p$ to within some tolerance, and $g_m$ are the modal coefficients, similar to $a_i$ and $d_i$. The newly produced orthogonal mode set, $\theta$, is also orthogonal to the $\psi$-modes contained in the set $\{ \psi_i \in i \rightarrow N_{REF} \}$. Finally, the unsteady surface pressure reconstruction for the candidate geometry becomes

$$
C_{CT}^*(\xi, \alpha, \dot{\alpha}; \alpha_0, k) = N_{REF} \sum_i d_i^{CT}(\alpha, \dot{\alpha}; \alpha_0, k) \psi_i^{REF}(\xi) + N_{CT} \sum_m g_m(\alpha, \dot{\alpha}; \alpha_0, k) \theta_m(\xi). \tag{4.14}
$$
4.8.1 Unsteady pressure field calculation

The split modes, $\theta$-modes, are first calculated using the same unsteady pressure field for the candidate airfoil as was used for the $\psi$-mode projections. The mode shape and corresponding modal coefficient for the first $\theta$-mode are shown in Figure 4.18 where the same pitch trajectory cases are considered as previously. The modal coefficient, $g_1$, is shown as a function of stall penetration angle, $\alpha_{sp}$, exhibiting a nonlinear response once the static stall angle of attack, $\alpha_{sp} = 0^\circ$, is exceeded. This mode shape appears to provide significant corrections to the leading edge region when the airfoil is positioned at higher angle of attacks.

Following the analysis of the previous sections, the aerodynamic loads are reconstructed using the five reference $\psi$-modes plus the integrated values of the first split mode. The normal force and quarter-chord moment for this reconstruction are shown in Figure 4.19 where $C_n^*$ is given on top and $C_m^*$ on bottom. The ordinate label is the same as before, giving the summation of modes used to calculate the respective load. The experimental data is shown in black with error bars representing the standard deviation. The five-mode reconstruction using only the reference $\psi$-modes is shown as a dotted line while the current reconstruction with the addition of the first split mode is shown as a solid line. It is immediately clear that with the addition of the...
split mode the approximation now fully captures the relevant pressure field dynamics in both the attached flow and stalled flow portions of the pitch cycle trajectories. It is also quite remarkable that only one $\theta$-mode is required for this correction. This first split mode, $\theta_1$, can then be thought of as a geometry correction mode.

4.8.2 Steady pressure field calculation

To calculate the $\theta$-modes described in the previous section requires the unsteady pressure fields of the candidate airfoil, making the model post-dictive in nature because the field being approximated has already been acquired experimentally. For this reason, it is desirable to be able to derive the split mode from the steady pressure field characterization of the candidate airfoil. Steady pressure field data is easily accessible, or at the least, easily acquired for any airfoil geometry. To calculate the
\( \theta \)-modes from steady data is the same process used for the unsteady data. That is,

\[
\tilde{C}_p^*(\xi, \alpha) = C_p^*(\xi, \alpha) - \sum_{i} d_{iT} (\alpha) \psi_{i\text{REF}}(\xi),
\]

(4.15)

where the modal coefficients, \( d_{iT} \), are the projection of the steady pressure field onto the \( \psi \)-modes. Once this operation is completed, the \( \theta \)-mode set is calculated as the PPOD of \( \tilde{C}_p^* \) as

\[
\tilde{C}_p^*(\xi, \alpha) = \sum_{m} p_m(\alpha) \theta_m(\xi),
\]

(4.16)

where \( p_m \) are the modal coefficients produced from the projection of the steady pressure field onto the \( \theta \)-modes. This orthogonal mode set, \( \theta \), calculated from the steady pressure field of the candidate airfoil is orthogonal to the \( \psi \)-modes contained in the set \( \{ \psi_i \rightarrow i \to N_{\text{REF}} \} \). The unsteady surface pressure reconstruction for the candidate geometry is then determined as before

\[
C_{pCT}^*(\xi, \alpha, \dot{\alpha}; \alpha_0, k) = \sum_{i} d_{iT} (\alpha, \dot{\alpha}; \alpha_0, k) \psi_{i\text{REF}}(\xi) + \sum_{m} g_m(\alpha, \dot{\alpha}; \alpha_0, k) \theta_m(\xi),
\]

(4.17)

where the unsteady \( \theta \)-mode coefficients \( g_m \) must be estimated.

The first \( \theta \)-mode and respective modal coefficients calculated from the PA1 steady pressure field is shown in Figure 4.20 superimposed on top of the unsteady pressure field result from Figure 4.18. Surprisingly, the steady mode shape looks identical to the unsteady mode shape, as no discernible difference can be seen between the two. This result is also supported by the modal coefficient response being nearly the same.
Figure 4.20. PA1 first split mode, $\theta_1$, and coefficient, $g_1$, using steady pressure field projection:

$\alpha_0 = 6.8^\circ$, $k = 0.020$ (—); $\alpha_0 = 10.5^\circ$, $k = 0.051$ (—); $\alpha_0 = 13.4^\circ$, $k = 0.078$ (—)

Figure 4.21 shows the reconstructions of $C_n^s$ and $C_m^s$ using the five reference $\psi$-modes and $\theta_1$ calculated from the PA1 steady pressure field. These reconstructions look identical to the ones produced from the unsteady $\theta-$modes, capturing all of the relevant details of the dynamic stall process, i.e. stall angles and magnitudes. This result is critical for the development of the dynamic stall model because it suggests that the steady pressure fields of the candidate airfoil can, indeed, be used to realize the geometry correction $\theta$-mode required for aerodynamic loading reconstruction. Also, the results indicate that the split mode, describing the introduction of physics due to geometric changes, is a steady characteristic.

4.9 Summary and conclusions

This chapter focused on the development of mode shapes for a low-order model from what was termed the “star” reference frame. Here, the unsteady pressure field around an arbitrarily chosen reference airfoil geometry is decomposed into locally optimal $\phi$-modes and globally optimal $\psi$-modes using classical POD and parametric POD (PPOD), respectively. The $\phi$-modes are parameter dependent spatial functions while the $\psi$-modes are purely spatial functions that are optimal over the entire range...
of the parameters space. When considering the aerodynamic loading reconstructions by spatially integrating the mode shapes, it is clear that the normal force repeatedly converges to the experimental data much quicker than the corresponding quarter-chord moment. This later convergence of the pitch moment is likely due to the fact that the moment calculation is more sensitive to aft pressure loadings, which fluctuate heavily during a dynamic stall event.

The primary take away of the reference airfoil analysis is that the parameter-invariant $\psi$-modes are able to adequately reconstruct the normal force and quarter-chord moment (to within $1\%–2\%$ of the respective deep stall amplitudes) across the parameter space with as few as 5 modes. This result demonstrates that a single set of spatial modes can describe the unsteady pressure field behavior across the entire
parameter space, making them an ideal candidate for a low-order model.

The latter half of the chapter focused on the reconstruction of candidate airfoil unsteady pressure fields and their resulting aerodynamic loads. The results presented show that large errors exist for the aerodynamic loads in the stalled regions of both the light and deep stall cases when the pressures are reconstructed using the reference basis.

To correct for these large discrepancies, an additional modal basis, termed $\theta$-modes, that is orthogonal to the reference basis, $\psi$-modes, is determined. This new modal set is calculated using split POD. It was determined that only the first mode from the $\theta$-mode basis is required to correct the aerodynamic load reconstruction to match the experimental data. The most important take away, however, is that the $\theta$-mode basis can be calculated using the steady pressure field characterization of the candidate airfoil. This result means that no unsteady testing is required to approximate the unsteady aerodynamic loads of a candidate airfoil, so long as the all of the modal coefficients can be well estimated.
CHAPTER 5

HAT METHOD PPOD CALCULATION

The second method considered for the process of developing a low-order dynamic stall model calculates a modal basis from the surface pressure field associated only with the unsteady behavior of the airfoil. Unlike the previous calculations, this method removes the entire steady pressure field, viscous and inviscid, from the experimentally obtained unsteady surface pressures. As will be shown, this method avoids the necessity for any type of split mode calculation to account for geometric changes.

5.1 “Hat” reference frame

As a reminder, Eq. 3.3 describing the “hat” reference frame is reproduced here

\[ \hat{C}_p(\xi, \alpha, \dot{\alpha}) = C_{p,\text{unsteady}}(\xi, \alpha, \dot{\alpha}) - C_{p,\text{steady}}(\xi, \alpha), \]  

(5.1)

where \( C_{p,\text{steady}} \) is the surface pressure field obtained from a steady characterization of an airfoil. Figure 5.1 shows an example of this reference frame methodology for the integrated normal force values of a NACA 23012 light stall case. One subtly that should not be dismissed in this calculation is that the steady pressure field values must be interpolated onto the angle of attack values for the unsteady case. For this reason, only unsteady cases with a maximum angle of attack less than or equal to the maximum angle of attack acquired in the steady characterization will be considered in order to avoid extrapolation of the steady pressures in a stalled regime. This
requirement limits the cases considered in this analysis to attached flow pitching and light stall.

![Graph of $C_n(\alpha, \dot{\alpha})$, $C_{n,\text{unsteady}}(\alpha, \dot{\alpha})$, and $C_{n,\text{steady}}(\alpha)$](chart.png)

Figure 5.1. “Hat” method for NACA 23012 $C_n$ values

5.2 $\phi$-mode calculation

As was done in Chapter 4, each independent parameter case, now only attached flow pitching and light stall, for a reference geometry is decomposed into parameter-dependent, spatial $\phi$-modes. The reference geometry is arbitrarily chosen to be the PA2 airfoil. Each independent parameter case considered for the PA2 is listed in Table 5.1 where entries corresponding to deep stall in Table 4.1 have been removed.

Figure 5.2 shows the first five eigenvalues corresponding to $\phi$-mode for each of the cases considered. The columns are organized by reduced frequency with the abscissa given as the mean angle of attack. The ordinate is log-scale to demonstrate...
TABLE 5.1

PA2 PARAMETER CASES FOR “HAT” METHOD ($\alpha_1 = 8^\circ$ FOR ALL CASES)

<table>
<thead>
<tr>
<th>Case #</th>
<th>$\alpha_0$</th>
<th>$k$</th>
<th>Type</th>
<th>Case #</th>
<th>$\alpha_0$</th>
<th>$k$</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$7^\circ$</td>
<td>0.020</td>
<td>AF</td>
<td>10</td>
<td>$10^\circ$</td>
<td>0.020</td>
<td>LS</td>
</tr>
<tr>
<td>2</td>
<td>$7^\circ$</td>
<td>0.050</td>
<td>AF</td>
<td>11</td>
<td>$10^\circ$</td>
<td>0.050</td>
<td>LS</td>
</tr>
<tr>
<td>3</td>
<td>$7^\circ$</td>
<td>0.0757</td>
<td>AF</td>
<td>12</td>
<td>$10^\circ$</td>
<td>0.0757</td>
<td>LS</td>
</tr>
<tr>
<td>4</td>
<td>$8^\circ$</td>
<td>0.020</td>
<td>LS</td>
<td>13</td>
<td>$11^\circ$</td>
<td>0.020</td>
<td>LS</td>
</tr>
<tr>
<td>5</td>
<td>$8^\circ$</td>
<td>0.050</td>
<td>LS</td>
<td>14</td>
<td>$11^\circ$</td>
<td>0.050</td>
<td>LS</td>
</tr>
<tr>
<td>6</td>
<td>$8^\circ$</td>
<td>0.0757</td>
<td>LS</td>
<td>15</td>
<td>$11^\circ$</td>
<td>0.0757</td>
<td>LS</td>
</tr>
<tr>
<td>7</td>
<td>$9^\circ$</td>
<td>0.020</td>
<td>LS</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>$9^\circ$</td>
<td>0.050</td>
<td>LS</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>$9^\circ$</td>
<td>0.0757</td>
<td>LS</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 5.2. PA2 $\phi$–mode eigenvalues, $\lambda_i$ as a function of mean angle of attack. Note $\lambda$ is non-dimensional.

the range of magnitude of the eigenvalues. Similar to the “star” method results in Figure 4.3, nearly all of the modal energy is carried by mode 1.
An example of the resulting mode shapes are shown in Figure 5.3 where the first five modes are shown for attached flow pitching (case #1) and light stall (case #15). It is interesting to note that $\phi_1$ and $\phi_3$ do not look too dissimilar from their “star” method counterparts in Figure 4.4. The modal coefficients, $a_i$, for each corresponding “hat” method case are shown in Figure 5.4. Not surprisingly, the attached flow modal coefficient behavior does not deviate very much from 0, indicating very little change from the steady values. However, the light stall case shows significant modal behavior all the way to mode 5.

Figure 5.3. PA2 $\phi$-modes for attached flow pitching and light stall
5.3 $\psi$-mode calculation

While the $\phi$-mode behavior is interesting, it describes the local parameter space behavior, leaving a global description to be desired. Using the same procedure described in §4.4, the local “hat” method $\phi$-modes are transformed to global $\psi$-modes, which are solely spatial functions. Figure 5.5 shows the resulting $\psi$-mode set for $i = 1, ..., 5$ along with the corresponding modal coefficients.
Figure 5.5. PA2 $\psi$-mode shapes and coefficients for attached flow pitching (——) and light stall (——).
Note that the ordinate scale of $d_1$ is nearly an order of magnitude larger.

5.4 $\psi$-mode reconstruction

As has been done in previous chapters, the integrated modal reconstructions of the normal force and quarter-chord pitching moment are used to distinguish the accuracy of the $\psi$-mode reconstructions.

Figure 5.6 and Figure 5.7 show the qualitative behavior of the normal force and the quarter-chord pitching moment, respectively, for an attached flow pitching case and a light stall case. Likewise, Figure 5.8 shows the light stall case reconstructions.
for both aerodynamic loads with the static values added to demonstrate that the actual loads can be recovered. Each figure shows the incremental addition of the aerodynamic loads calculated from the first five modes, $\psi_1 \rightarrow \psi_5$. The experimental data is shown in black with error bars representing the standard deviation of the force at the given pitch motion location. Likewise, the reconstructed values are shown in
Remarkably, the normal force is almost entirely captured by only the first mode. The addition of subsequent modes appear to only slightly adjust the stalled portions of the pitching cycle. By mode 5 the reconstruction is entirely contained within the standard deviation bounds for nearly the whole pitch cycle for the light stall case.
quarter-chord moment reconstruction shows more sensitivity to the addition of each mode, at least in the light stall case. The attached flow pitching case is adequately reconstructed with only one mode, however, the light stall case requires a few more. The characteristics of the experimental data, describing the moment stall and peak nose-down moment, begin to be captured with the addition of mode 3. Subsequently, the additions of mode 4 and mode 5 adjust the magnitudes at these characteristics to match the experimental data. It is likely that the 5th mode is more important to a deep stall case as was seen in the “star” method in Chapter 4.

The quantitative errors, as calculated in Eq. 4.7, for the normal force and the quarter-chord pitching moment are presented in Figure 5.9. Here, the errors rapidly decrease within the first five modes for both loads. Comparing the error magnitudes for a five-mode reconstruction of each load to the respective amplitude range of the experimental light stall case, being 1.37 for the normal force and 0.23 for the quarter-chord pitching moment, results in the following error percentages:

\[ \hat{C}_n \text{ five-mode reconstruction} \]
- Attached flow pitching: 0.9%,
- Light stall: 1.8%,

\[ \hat{C}_m \text{ five-mode reconstruction} \]
- Attached flow pitching: 0.9%,
- Light stall: 2.5%.

This quantitative error convergence and the qualitative behavior of the reconstructions exhibited in Figure 5.6 and Figure 5.7 suggest that a five-mode reconstruction at each parameter point, \((\alpha_0, k)\), is more than adequate for capturing the dynamics associated with the time evolution of the unsteady surface pressure field of the reference PA2 airfoil.
5.5 Projection onto the reference $\psi$-mode basis

Continuing with the same analysis as was considered for the “star” method in Chapter 4, the modal coefficients for candidate airfoils are calculated and compared for the “hat” method by projecting the unsteady surface pressure field, $\hat{C}_p$, of candidate airfoils onto the reference modal basis. Likewise, aerodynamic loads, normal force and quarter-chord pitching moment, are reconstructed for the candidate airfoils to determine the need of split modes. The surface pressure projection for a candidate...
The first five modal shapes, derived in the previous chapter, are shown in Figure 5.10 along with the projected modal coefficients, $d_i$, for the reference airfoil (PA2) and two sample candidate airfoils (PA1 and NACA 23012). For each airfoil an attached flow case and a light stall case is shown.

Unlike the modal coefficients determined for the “star” method, these coefficients tend to oscillate around a 0 zero value. Although the modal coefficients do exhibit dependencies on airfoil geometry, their behaviors are somewhat similar. The amplitude
of the coefficients tends to increase at angles of attack beyond the static stall angle, $\alpha_{ss} = 0^\circ$, which is not surprising considering the formulation of $\hat{C}_p$. However, mode 1 and mode 3 show modal influence at angles less than the static stall angle suggesting that these modes are responsible for the delay in boundary layer reattachment seen on the downstroke of a light stall pitching trajectory.
5.6 Candidate geometry $\psi$-mode reconstruction

To access the ability of the reference $\psi$-mode basis to fully capture the dynamics of the unsteady surface pressure of sample candidate airfoils, reconstructions of the aerodynamic loads are considered for both the PA1 geometry and the NACA 23012. The normal force and quarter-chord moment reconstructions for the PA1 geometry are shown in Figure 5.11 and Figure 5.12 respectively.

It is immediately recognizable that the normal force is very well approximated by the reference basis for both the attached flow case and the light stall case using a five-mode reconstruction. The attached flow case appears to be fully realized by the addition of mode 3. On the other hand, the light stall case is nearly captured over the entire pitching cycle by the addition of mode 5. The only part of the approximation not contained within the bounds of the experimental standard deviation limits is near the peak value.

For the PA1 quarter-chord pitching moment, the five-mode approximation also does an excellent job describing its behavior for both attached flow pitching and light stall. While the peak nose-down moment of the light stall case is not fully realized, all of the remainder of the cycle appears to be within the standard deviation of the experimental results.

When comparing the error magnitudes for a five-mode reconstruction of each load to the respective amplitude range of the experimental light stall case, being 1.52 for the normal force and 0.23 for the quarter-chord pitching moment, results in the following error percentages:

\[ \hat{C}_n \text{ five-mode reconstruction} \]
- Attached flow pitching: 1.4%,
- Light stall: 4.9%,
\[ \alpha_0 = 6.8^\circ, k = 0.020 \]

\[ \alpha_0 = 10.5^\circ, k = 0.077 \]

Figure 5.11. PA1 normal force reconstruction, \( \hat{C}_n \), from projected \( \psi \)-mode: attached flow pitching and light stall

\( \hat{C}_m \) five-mode reconstruction

- Attached flow pitching: 2.6%,
- Light stall: 7.6%.

While these errors are definitely higher than those for the reference geometry reconstructions, they are the same order of magnitude. These results suggest that a five-mode reconstruction captures the dynamics of the candidate PA1 unsteady
surface pressure field.

To further validate this conjecture that the reference modes are sufficient for fully describing the candidate airfoil’s surface pressure behavior, another candidate airfoil is considered. The normal force and quarter-chord moment for the NACA 23012 are shown in Figure 5.13 and Figure 5.14 respectively. Once again, the five-
Figure 5.13. NACA 23012 normal force reconstruction, $\hat{C}_n$, from projected $\psi$-mode: attached flow pitching and light stall mode approximation fully resolves the normal force and quarter-chord moment curves for both the attached flow and light stall cases despite the apparent differences in behavior when compared to the PA1 loads. This result suggests that the reference $\psi$-mode basis is a general basis for describing the unsteady surface pressure, $\hat{C}_p$, across arbitrary airfoil geometries.
The error percentages for a five-mode reconstruction of each load calculated from the respective amplitude range of the experimental light stall case, which is 1.30 for the normal force and 0.22 for the quarter-chord pitching moment, are listed in the following:

\[ \hat{C}_n \text{ five-mode reconstruction} \]
- Attached flow pitching: 1.5%,
- Light stall: 2.2%,

\( \dot{C}_m \) five-mode reconstruction
- Attached flow pitching: 1.7%,
- Light stall: 6.3%.

These error percentage values are very similar to those determined for the PA1 projections, which are also comparable to those determined for the reference PA2 airfoil.

5.7 Summary and conclusions

This chapter focused on the decomposition of the purely unsteady pressure field, \( \dot{C}_p \), around an arbitrarily chosen reference airfoil geometry into \( \phi \)-modes and \( \psi \)-modes using classical POD and parametric POD (PPOD), respectively. Again, the \( \phi \)-modes are parameter dependent spatial functions while the \( \psi \)-modes are purely spatial functions that are optimal over the entire range of the parameters space. The primary conclusion of this chapter is that the parameter-invariant \( \psi \)-modes are able to adequately reconstruct the normal force and quarter-chord moment (to within 1%–2% of the respective light stall amplitudes) across the parameter space with as few as 5 modes. This result demonstrates that a single set of spatial modes can describe the unsteady pressure field behavior across the entire parameter space, making them an ideal candidate for a low-order model, as was demonstrated for the “star” method.

The latter half of this chapter considered the projection of multiple candidate airfoil geometries, namely the PA1 airfoil and the NACA 23012, onto a reference \( \psi \)-mode basis determined from the PA2 airfoil. The resulting modal coefficients showed similar behaviors across the geometries for both the attached flow pitching and light stall cases. However, the most notable conclusion determined in this chapter
is that reference $\psi$-mode shapes determined in the “hat” reference frame are able to reconstruct the unsteady pressure field behavior of candidate airfoils without the requirement of an additional orthogonal basis, such as the split mode basis derived for the “star” method.
Now that the parameter-invariant $\psi$-mode shapes have been calculated for both the “star” and “hat” methods it seems appropriate to examine the physical behavior of the modes contributing the most to the aerodynamic load reconstructions. For this analysis, only the mode shapes and corresponding modal coefficients provided from the reference PA2 airfoil geometry will be considered.

6.1 $\psi$-mode shape contributions

Using Eq. 4.5 and Eq. 4.6 as reference, the individual modal contributions to the normal force and quarter-chord pitching moment, respectively, can be calculated to determine the sensitivity of each aerodynamic load to the respective mode shape. This process is completed by separating the equations into modal coefficients and integrated shape contributions as follows:

\[
C_n(\psi_i) = d_i \int_0^1 \left( \psi_i^+(\xi) - \psi_i^-(\xi) \right) d\xi, \hspace{1cm} (6.1)
\]

\[
C_m(\psi_i) = d_i \left( \frac{1}{4} C_n^*(\psi_i) - \int_0^1 \left( \psi_i^+(\xi) - \psi_i^-(\xi) \right) \xi d\xi \right), \hspace{1cm} (6.2)
\]

where $C_n^*(\psi_i)$ and $C_m^*(\psi_i)$ are the integrated mode shape contributions to the normal force and pitching moment, respectively. Essentially, the mode shape contribution is
a scaling parameter for the modal coefficient, $d_i$. Therefore, the functional shape of $C_x(\psi_i)$ is defined by the modal coefficient, $d_i$, while the magnitude of this function is controlled by the mode shape contribution, $C'_x(\psi_i)$. In general, the magnitude of the modal coefficients decreases when moving to higher order modes due to the POD calculation forcing the mode shapes to be orthonormal, thus, requiring the amplitudes of their respective modal contributions to be employed by the modal coefficients. However, this decrease in magnitude with increasing mode number cannot be assumed for the aerodynamic loads because of the scaling introduced by the mode shape contribution, which will be shown in the following.

The values of the PA2 $\psi$-mode shape contributions to the normal force and the quarter-chord pitching moment for both the “star” and “hat” methods are given in Table 6.1 for modes $\psi_1 \rightarrow \psi_5$. It is clear that there is no preference in the ordering due to mode number. To supplement the results here, Figure 6.1 and Figure 6.2 are provided to show the calculation of the modal contribution to each aerodynamic load for the “star” method and the “hat” method, respectively. In these figures the load shapes, which are defined as $\Delta \psi_i(\xi) = \psi^+_i(\xi) - \psi^-_i(\xi)$, along with the corresponding modal coefficients are on the left side. The right side gives the modal contribution to the aerodynamic loads, $C_x(\psi_i)$, which are obtained by appropriately integrating the load shapes and multiplying by the modal coefficients as previously discussed. The modal coefficients and modal contributions are given as functions of non-dimensional time, $t/T$, where $T$ is the period of the pitching cycle.

It is immediately obvious that the resulting aerodynamic load contributions, $C_x(\psi_i)$, are not ordered by decreasing amplitude, as the modal coefficients are, for the “star” method or the “hat” method as evident from the large amplitudes seen for $C_m(\psi_3)$. These contributions, also, do not follow the same ordering suggested in Table 6.1 which says that the modal coefficient magnitudes and the mode shape...
TABLE 6.1
PA2 $\psi$-MODE SHAPE CONTRIBUTIONS

<table>
<thead>
<tr>
<th>Mode #</th>
<th>$C_n^*(\psi_i)$</th>
<th>$C_m^*(\psi_i)$</th>
<th>$\hat{C}_n^*(\psi_i)$</th>
<th>$\hat{C}_m^*(\psi_i)$</th>
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<td>0.136</td>
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<td>4</td>
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<td>5</td>
<td>0.57</td>
<td>-0.177</td>
<td>0.06</td>
<td>-0.117</td>
</tr>
</tbody>
</table>

Contribution magnitudes must be considered together. Ultimately, these results indicate that while the calculated $\psi$-modes may be optimal for the surface pressure field reconstruction, they are not for the reconstruction of the aerodynamic loads. This non-optimal behavior is further supported by the fact that the $\Delta \psi$ load shapes used to calculate the aerodynamic loads can no longer be assumed to be orthonormal functions of space like the $\psi$-modes.

While these shapes may not be linearly independent from one another, physical insights can still be gleaned regarding the behavior of the dynamic stall process. For this analysis, each method, “star” and “hat”, will be considered independently.

6.2 “Star” method coefficient behavior

In this analysis, the loads contributing the most to the aerodynamic loads will be considered and physical significance of these modes will be inferred. Due to their lack of significant contribution to the aerodynamic loads, mode 4 and mode 5 will be excluded from this analysis.
6.2.1 Mode 1

Looking again at Figure 6.1 it is well demonstrated that mode 1 heavily influences both aerodynamic loads. The functional shape of $d_1$ when viewed as a function of $t/T$ appears to be nearly self-similar, with the magnitude of the shape controlled by either the mean angle of attack, $\alpha_0$, or the reduced frequency, $k$. Figure 6.3
Figure 6.2. PA2 “hat” method load contribution calculation:
attached flow pitching (---) and light stall (—)

shows the maximum value of $d_1$ as a function of maximum stall penetration angle,
$\alpha_{sp\mid max} = \alpha_0 + \alpha_1$. Remarkably, a linear relationship with maximum angle of attack is realized once the stall penetration is high enough, which is speculated to coincide with the location leading edge vortex production. The light and deep stall regimes, $\alpha_{sp\mid max} \sim 4^\circ$, are invariant to reduced frequency. However, the attached flow
pitching and stall onset regimes, $\alpha_{sp}\big|_{\text{max}} < \sim 4^\circ$, show a small dependence on reduced frequency. This result suggests that modeling mode 1 depends on determining a functional shape for the mode as a function of $t/T$ and then scaling this function based on the maximum stall penetration angle.

The question to be asked now is, does this same linear relationship hold for the mode 1 coefficients of the candidate airfoils that were determined by modal projection? Figure 6.4 shows the maximum value of $d_1$ for each airfoil, where the PA2 is the reference airfoil and the PA1 and NACA 23012 are the candidate airfoils. It is clear that the linear relationship with maximum stall penetration angle remains; however, the slope of the curve appears to have a geometric dependence.

Physically, mode 1 represents the reduction in suction pressure occurring in the
leading edge region of the airfoil for the experimental values compared to the results of the panel method solution. The modal coefficients support this claim because the load shape, indicating the loss of leading edge suction, is amplified as the airfoil pitches up ($t/T < 0.5$) and is then reduced as it begins to pitch down ($t/T > 0.5$).

6.2.2 Mode 2

The modal coefficients for the second “star” method mode, $d_2$, shown in Figure 6.1 do not exhibit a similar functional shape across the different regimes as was seen for mode 1. However, trends for the maximum and minimum values do exist as shown in Figure 6.5 and Figure 6.6 respectively. The maximum values show an invariance to reduced frequency until a maximum stall penetration angle of about $5^\circ$, where the curves then bifurcate into branches corresponding to reduced frequency. This bifurcation angle is higher than the pivotal stall penetration angle of $4^\circ$ that was seen for the maximum mode 1 values, suggesting the influence of higher strength
vortices. On the other hand, the minimum values shown in Figure 6.6 show a mild invariance to reduced frequency. With the exception of the attached flow values hovering around $d_2|_{\text{min}} = -0.1$, the minimum values are well behaved, showing a nearly constant magnitude.

![Figure 6.5. PA2 “star” method $d_2$ maximum amplitude](image)

A physical interpretation of mode 2 is not as straightforward as mode 1. Looking at its contribution to the aerodynamic loads, this mode primarily affects the normal force during the stalled portion of the pitching cycle. When large vortices are realized, as in the deep stall case, this mode assists in augmenting the normal force, which is attributable to the maximum values of $d_2$. However, the minimum values look to contribute to the delay in reattachment that is seen with increasing reduced frequency and mean angle of attack.
6.2.3 Mode 3

As with mode 1, the modal coefficients for the third mode, $d_3$, in Figure 6.1 show a self-similar behavior for the light and deep stall regimes. Following the same logic as before, the maximum values should behave as scaling parameters. These values are given in Figure 6.7. Interestingly, there is a strong dependence on reduced frequency once leading edge vortices begin to form, which from this data occurs at $\alpha_{sp|\text{max}} \approx 4^\circ$. Delays due to reduced frequency can be seen at maximum stall penetration angles less than $4^\circ$, where the different curves jump to there higher values at a higher angle as reduced frequency increases.

Clearly, this mode is related directly to the dynamic stall vortex propagation due to its extreme aft loading, which is demonstrated in the load shape. As a result, this mode is a larger contributor to the quarter-chord pitching moment, having an amplitude on the same order of magnitude as that contributed by mode 1. While the amplitudes of this mode are not as clearly estimated as those for mode 1, they still
display a nice functional relation. If the maximum values of mode 1 and mode 3 could be consistently approximated then the maximum nose-down pitching moment would be very well estimated, which is of crucial importance for engineering purposes.

6.3 “Hat” method coefficient behavior

This same analysis can also be completed for the “hat” method, where the mode load shapes describe the pressure loading behavior relative to the steady, $\dot{\alpha} = 0$, values. Although mode 4 contributes to the quarter-chord pitching moment loading, only modes 1-3 will be examined here.

6.3.1 Mode 1

The load shape for mode 1 shown in Figure 6.2 looks very similar to the one determined for the “star” method. This load shape is a leading edge modification describing the suction peak behavior of the airfoil and is the largest contributor to
the normal force loading. The functional shape of \( d_1 \) when viewed as a function of \( t/T \) shows that the suction peak grows in magnitude for the light stall case in the range \( t/T \in [0.30.5] \), which is supported by the increase observed in the normal force contribution for this time range. This growth indicates the delay in stall due to a positive pitch rate, which is supported in Figure 5.5 where the negative portion of \( d_1 \), indicating an increase in suction pressure, occurs after \( \alpha_{sp} = 0^\circ \). Figure 6.8 shows the minimum value of \( d_1 \) as a function of maximum stall penetration angle, \( \alpha_{sp}|_{max} \). It is evident that the amplitude of this suction pressure augmentation is dependent on maximum stall penetration angle and the reduced frequency. This figure suggests that suction pressure amplitude increases with increasing stall penetration angle. However, assuming that the leading edge vortex becomes significant around \( \alpha_{sp}|_{max} = 3^\circ \), reduced frequency effects become important in defining the suction peak amplitude.

The pitch rate induced delay in maximum suction peak amplitude is demonstrated in Figure 6.9. Here, the peak value tends to occur earlier in the pitching cycle for all reduced frequencies as the maximum stall penetration angle is increased. However, the effect of reduced frequency is seen to delay this peak value to later times in the pitching cycle for a given stall penetration angle.

6.3.2 Mode 2 and mode 3

Mode 2 and mode 3 are discussed together because of their collective contribution to what is the maximum nose-down quarter-chord pitching moment seen in Figure 6.2. Because of their load shapes, it appears that these two modes are similar to a conjugate pair describing the evolution of the dynamic stall vortex, which is the mechanism that induces the large nose-down moment. The load shapes show an opposing behavior between the two modes; however, the modal coefficients also act in opposite directions, suggesting that these two modes actually influence the loading
Figure 6.8. PA2 “hat” method $d_1$ minimum amplitude

Figure 6.9. PA2 “hat” method $t/T(d_1|_{min})$ values
in the same manner, which is demonstrated by their respective quarter-chord moment contributions.

Figure 6.10 and Figure 6.11 show the minimum $d_2$ values and the maximum $d_3$ values as a function of maximum stall penetration angle, respectively. It appears that the magnitude of the $d_2$ peak increases regularly with increasing stall penetration angle. Likewise, the magnitude of the $d_3$ peak increases with increasing stall penetration angle with the exception of the lowest reduced frequency values. The combined effect of these behaviors suggests an increase in the magnitude of the dynamic stall vortex with increased stall penetration angle.

Due to the lack of deep stall cases, it is difficult to determine if the functional shapes of $d_2$ and $d_3$ is self-similar as was observed for the “star” method modes related
Figure 6.11. PA2 “hat” method $d_3$ maximum amplitude

to the dynamic stall vortex. However, it does appear that the maximum nose-down quarter-chord moment can be appropriately predicted if the peak values of $d_2$ and $d_3$ can be faithfully approximated.

6.4 Summary and conclusions

This chapter considered the physical implications of the mode shapes provided by the “star” method and the “hat” method. It is shown that while the mode load shapes discussed may not be linearly independent, they still provide insight concerning the physical mechanisms that they describe. The primary contributing modes of both the “star” method and the “hat” method when the aerodynamic loads are concerned describe the leading edge suction peak behavior and the evolution/propagation of the dynamic stall vortex. It is also shown that with the approximation of the some of the modal coefficient peak values, important quantities can be predicted, such as the maximum nose-down pitching moment.
CHAPTER 7

CONCLUSIONS AND FUTURE WORK

This dissertation demonstrates the derivation of spatial modes to supplement a low-order unsteady surface pressure model describing the effects of the dynamic stall phenomenon. The spatial modes are calculated for an arbitrary reference airfoil geometry using a parametric proper orthogonal decomposition, allowing them to operate optimally over a parameter space. The parameter space considered in this study encompasses airfoil pitching trajectories in the unsteady attached flow, light stall, and deep stall regimes. This type of range includes multiple mean angles of attack and reduced frequencies. Two frames of reference are considered for the low-order model development: the “star” reference frame and the “hat” reference frame.

7.1 “Star” method results

The “star” reference frame considers the unsteady surface pressure field sans the steady, inviscid pressure field, supplied from a panel method solution, leaving only the viscous and unsteady surface pressure. Important results from analyzing this type of pressure field include:

- For an arbitrarily chosen reference airfoil, the aerodynamic loads (normal force and quarter-chord pitching moment) are adequately reconstructed with as few as 5 $\psi$-modes. The associated error of the aerodynamic load reconstructions using the parameter-invariant $\psi$-modes are the same order of magnitude as those calculated from the corresponding parameter-dependent $\phi$-mode reconstructions.

- The five-mode reference $\psi$-mode basis is determined to not be adequate for consistently and fully reconstructing the unsteady surface pressures of a candidate
airfoil.

- To fully reconstruct the candidate airfoil surface pressures, an additional modal basis, termed $\theta$-modes, is derived by the application of split mode proper orthogonal decomposition. Only the first mode, $\theta_1$, is required for the reconstruction.

- It is determined that this $\theta$-mode set can be calculated from either the unsteady or steady surface pressure field of the candidate airfoil, making it suitable for application in a low-order model.

7.2 “Hat” method results

Similar to the “star” reference frame, the “hat” reference frame also remove steady information from the surface pressure field. However, in this frame only the unsteady contributions to the surface pressure field are considered, which is achieved by removing the steady, viscous pressure field. Important results from this reference frame include:

- For an arbitrarily chosen reference airfoil, the aerodynamic loads are adequately reconstructed with as few as 5 $\psi$-modes.

- Unlike the “star” reference frame method, the five-mode $\psi$-mode basis also describes the surface pressure response of multiple candidate airfoils. Therefore, no additional modes, such as those from a split mode derivation, are required to capture any additional physics introduced by a geometric change of the airfoil.

The final result of the “hat” reference frame method suggests a very subtle behavior of the dynamic stall phenomenon. This result says that the differences in the aerodynamic loads produced by different airfoil geometries are manifest by the steady, viscous pressures associated with the geometry. This leaves the unsteady effects of dynamic stall to be general perturbations around this steady, viscous behavior. Perhaps this conclusion is shown more clearly by considering the relationship between the two reference frames:

$$C_p^*(\xi, \alpha, \dot{\alpha}; \alpha_0, k) = C_p(\xi, \alpha, \dot{\alpha}; \alpha_0, k) + C_{p,visc}(\xi, \alpha).$$

(7.1)
Substituting in the reconstruction for the reference geometry yields

\[
N_{\text{REF}} = 5 \sum_{i=1}^{N_{\text{REF}}=5} \hat{d}_{i}^{{\text{REF}}} \hat{\psi}_{i}^{{\text{REF}}} + C_{p,\text{visc}}(\xi, \alpha) = \sum_{i=1}^{N_{\text{REF}}=5} d_{i}^* \psi_{i}^*.
\]  

(7.2)

The conclusion becomes obvious once the candidate geometry reconstruction is applied,

\[
N_{\text{REF}} = 5 \sum_{i=1}^{N_{\text{REF}}=5} \hat{d}_{i}^{{\text{CT}}} \hat{\psi}_{i}^{{\text{REF}}} + C_{p,\text{visc}}(\xi, \alpha) = \sum_{i=1}^{N_{\text{REF}}=5} d_{i}^{{\text{CT}}} \psi_{i}^{{\text{REF}}} + g_{1} \theta_{1}^{{\text{CT}}}.
\]  

(7.3)

Although the modal coefficients do exhibit geometric dependencies, the dynamics associated with the reference and candidate airfoils are realized using the same modal basis within the “hat” reference frame method, as shown in Chapter ???. Therefore, Eq. 7.3 says that the difference in \( C_{p,\text{visc}}(\xi, \alpha) \) between the reference and candidate geometry results in the need for the additional \( \theta \)-mode set.

7.3 Suggestions for future work

The most obvious task that remains for the low-order model development is a calculation/estimation procedure for the modal coefficients given an arbitrarily defined pitching trajectory. Currently, the modal coefficients have been cast into a dynamical system where the derivative of each coefficient with respect to time is estimated as a polynomial expansion of state variables, which are composed of the geometric coefficients, modal coefficients, and motion parameters such as angle of attack, pitch rate, and pitch acceleration. The relative contribution of each term in the expansion is then determined by a least-squares approximation. The result is a system of ordinary differential equations that can be integrated in time given an initial state [10]. Another promising procedure estimates a dynamical system of the modal coefficients using a linear-parameter-varying model [22, 33] with inputs consisting of the airfoil motion state, \( i.e. \) angle of attack and pitch rate.
Other tasks that should be explored using this model development include studying the dynamics of the surface vorticity flux due to each mode. This type of analysis was introduced by Reynolds and Carr [45], which relates the spatial derivative of the surface pressure to vorticity flux. Using this relation, the contribution of each mode to the local unsteady vorticity flux can be determined and ultimately elucidate additional physics associated with dynamic stall inception.

The final recommendation is for time-resolved particle image velocimetry to be simultaneously acquired with the unsteady surface pressure of an airfoil undergoing pitching trajectories similar to those studied in this work. The resulting velocity field should be conditionally sampled by the mode shapes describing the surface pressure. This conditionally sampled velocity field can then describe the external flow field structures that are associated with the surface pressure modes of the low-order model providing further physical insight to the pressure mode shapes.
## APPENDIX A

### PROPRIETARY AIRFOIL PRESSURE PORT DETAILS

#### TABLE A.1

**PA1 PRESSURE PORT DETAILS**

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<th>Chord Station</th>
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<th>Pressure Sensor</th>
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### TABLE A.2

**PA2 PRESSURE PORT DETAILS**

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Figure B.1. NACA 23012 normal force reconstruction, $C_n^*$, from projected $\psi$-mode: attached flow pitching, light stall, and deep stall
$\alpha_0 = 7.7^\circ, k = 0.051$

$\sum_{i=1}^{5} C_m(\psi_i)$

Experimental POD reconstruction

Figure B.2. NACA 23012 quarter-chord moment reconstruction, $C_m^*$, from projected $\psi$-mode: attached flow pitching, light stall, and deep stall
Figure B.3. NACA 23012 first split mode, $\theta_1$, and coefficient, $g_1$:

- $\alpha_0 = 7.7^\circ$, $k = 0.051$ (= )
- $\alpha_0 = 10.7^\circ$, $k = 0.077$ (= )
- $\alpha_0 = 15.7^\circ$, $k = 0.104$ (= )

Figure B.4. NACA 23012 aerodynamic loads reconstructions from $\psi$-modes plus the first split mode, $\theta_1$: attached flow pitching, light stall, and deep stall.
Figure B.5. NACA 23012 first split mode, $\theta_1$, and coefficient, $g_1$ using steady pressure field projection:
$\alpha_0 = 7.7^\circ, k = 0.051$ ; $\alpha_0 = 10.7^\circ, k = 0.077$ ; $\alpha_0 = 15.7^\circ, k = 0.104$

Figure B.6. NACA 23012 aerodynamic loads reconstructions from $\psi$-modes plus the first split mode, $\theta_1$ calculated from the steady pressure field: attached flow pitching, light stall, and deep stall
BIBLIOGRAPHY


