RISK MANAGEMENT OF ENERGY COMMODITIES—VALUATION AND OPTIMIZATION OF
AN ENERGY SWAP FOR A FLEXIBLE FUEL PROCESS AND ENERGY COMMODITY STORAGE

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Abstract

Energy commodities have significant roles in modern industrial economics. Energy price fluctuations caused by unbalanced supply and demand and other exotic reasons can result in financial risks. To mitigate or avoid the financial risks, risk management is necessary and profitable for energy utility owner/operators. In this dissertation, two problems in energy risk management are considered. One is ‘valuation of an energy swap for a flexible fuel process’ and the other is ‘valuation and optimization of energy commodity storage’.

An energy swap is a contract between energy utility owner/operators and a banker, where the energy utility owner/operators pay the banker a certain amount of money initially and the banker will take care of the energy cost of the energy utility during an agreed period of time. The problem is to decide a ‘fair value’ for the energy utility owner/operator to pay. To obtain the ‘fair value’, a self-financing hedging portfolio is constructed consisting of a flexible fuel process, physical ownership of the underlying energy commodities whose stochastic price models are correlated, and a risk-free asset. PDE method is employed to obtain the valuation of the energy swap and
the optimal control strategies (the hedging) for the flexible fuel process. Both simulated data with Monte Carlo method and historical data are used for validation. The results can be used either by the process operator directly to reduce financial risk, or by an energy banker to price an 'energy swap' to finance process operation.

As for the storage problem, the purpose is to help energy utility with energy storage obtain the optimal storage operations to lower financial risks and to make more profit. First, a storage model for a single commodity with stochastic price serving a known and fixed demand is proposed. Convenience yield is also included in the model. A 2-dimensional (2D) Hamilton-Jacobi-Bellman (HJB) equation with embedded optimization problem is formulated and solved to obtain the optimal storage strategies. An oil storage problem is given as an example. The valuation result is compared to two other cases with different control methods and ends up performing the best, which demonstrates the benefit to operate the facility following the optimal control strategies.

The control problem is found to be a stochastic sliding mode control problem, which appears novel. The storage problem is further extended by including stochastic demand instead of known demand. A similar 3-dimensional (3D) HJB equation is formulated and solved. The results are consistent with the 2D’s. The convergence of the 3D computation is verified with the numerical convergence order. This extended 3D model can be easily adapted for other stochastic variable, like stochastic convenience yield, stochastic volatility or another stochastic commodity price. Proper boundary conditions need to be chosen to solve the corresponding 3D problem. This storage model can also be used for general commodity storage optimization problems.
This is for my family, who loves me and supports me so much.
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CHAPTER 1:
INTRODUCTION

Unbalanced supply and demand and other exotic reasons can cause energy commodity price fluctuate and result in financial risks for energy utility owner/operators. Effective energy risk management can help mitigate or avoid those risks and bring profits for the energy utility owner/operators. With these purposes, two problems in energy risk management are considered. One is ‘valuation of an energy swap for a flexible fuel process’ and the other is ‘valuation and optimization of energy commodity storage’.

In the first problem, a hedging portfolio including a flexible fuel process is employed to evaluate an “energy swap”. A partial differential equation (PDE) is obtained and solved for the valuation results and hedging strategies. The contributions we made are the inclusion of a flexible fuel process with a first law of thermodynamics consistent model in the hedging portfolio, compared to conventional formulations available in the literature [Eydeland2003, Geman2005, Benth2008]. So far, relatively few literatures are reported on ‘energy swap’. This project should be considered as a frontier study combing chemical process with financial instrument valuation. Besides, special
consideration is given to the necessary boundary conditions associated with the PDE and the inclusion of convenience yield.

As for the storage problem, a storage model including convenience yield and known/stochastic demand is constructed. A PDE named Hamilton-Jacobian-Bellman (HJB) equation with an embedded optimization problem is derived and solved with a finite difference method to obtain the optimal storage operation strategies. The optimal control problem is further found to be a stochastic sliding model control problem, which appears novel and worth further investigation.

This chapter will introduce some background knowledge about risk management and energy price modeling. It is organized as follows. In section 1.1, risk management and the importance of energy risk management are introduced. In section 1.2, the methods of risk management are discussed. Since risk management strategies are highly related with energy price change, energy commodity price modeling including price characteristics and price models is described in section 1.3.

1.1 Overview

1.1.1 What is risk management

Risk management is the identification, assessment, and prioritization of risks followed by coordinated and economical application of resources to minimize, monitor, and control the probability and/or impact of unfortunate events or to maximize the realization of opportunities [Hubbard2009]. Risk management is a central consideration of managing business including enterprise [Lam2014], medical device
[Burton2006], natural disaster [Smith2013], information technology [Schwalbe2013], and energy commodities [Chkili2014].

1.1.2 Why energy commodity risk management

Energy commodities like coal, natural gas, oil, are important for daily life and the global development. For example, we use natural gas for cooking, cooling in the summer and heating in the winter. The importance of energy commodity makes energy risk management essential and urgent. Due to price fluctuation caused by unbalanced supply and demand and other exogenous events, energy utility owner/operators suffer from potential risks. For example, if natural gas price soar extremely high, natural gas users need to pay high bills. Therefore risk management is necessary and helpful.

1.2 Risk management methods

There are three main strategies in energy risk management, financial instruments, physical storage, and development of flexible utility. Energy utility owner/operators use financial instruments to lock the price to prevent further potential risks. By treating physical storage as a cushion, energy utility owner/operators can buy energy commodity and store them when energy price is relatively low, sell them when price is relatively high to avoid the potential risks or even make more profits. With flexible fuel operation, energy utility owner/operators can avoid the risks caused by a single fuel. Since if that single fuel price increases sharply, the energy utility owner/operators have to pay high bills to buy it. Instead, by operating with alternative fuels, the energy utility owner/operators have more freedom to response to the
fluctuated prices of different fuels and avoid the financial risks by choosing the cheaper one. In this method, the energy utility owner/operators are still subject to the potential risks cause by the price fluctuation of no matter which fuel they pick. Therefore it is also combined with the first method. In this section, we will focus on the first two methods.

1.2.1 Hedging with financial instruments

1.2.1.1 Financial instruments

A financial instrument is a tradable asset of any kind, either cash or commodity or a contractual right to deliver or receive cash or another financial instrument. The commonly used financial instruments include forward, futures, swaps, options, etc.

1.2.1.1.1 Forward

A forward contract (or forward) is a non-standardized contract between two parties to buy or sell an asset at a specified future time at a certain price agreed today. Forward contract trades in the over-the-counter (OTC) market. (The OTC market is an important alternative to exchanges. It is a network of traders who work for financial institution, large corporations, or fund managers.)

Forward contracts are widely used in the electricity markets. For example, forwards are used for electricity supply and demand in developing countries like Brazil, Chile, etc. [Lima2014] Woo et al. developed a linear regression model for pricing the forward contracts in the presence of temperature and hydro risks against electricity spot-price risk in the Pacific Northwest [Woo2011]. Gökgöz and Atmaca did research on
electricity generation asset allocation between bilateral contracts, such as forward contracts, and daily spot market, considering constraints of generating units and spot price risks through Turkish electricity market. They found the optimal portfolio based on known electricity generation total costs, bilateral contract prices [Gökgöz2011].

Sotiropoulos et al. proposed an hourly load model for fundamental modeling of forward contract pricing for electricity in Europe. This model captured deterministic patterns such as yearly and weekly seasonality, intra-day patterns including holidays, temperature effects and economic trends. These pattern and trends can be estimated with computational efficiency via Maximum Likelihood Estimator [Sotiropoulos2013]. Forward contracts can be used to hedge foreign currency risk [Hodrick2014]. They can also be used for procurement strategies. Lee et al. used forward contacting and the spot market to formulate procurement strategies and determine the optimal procurement quantity in order to maximize profit. A case study in a steel making company in India was given with real data as an example [Lee2014].

1.2.1.1.2 Futures

A future contract (or future) is a standardized contract between two parties to exchange a specified asset of standardized quantity and quality for a certain price. Futures are traded on exchanges. A key difference between forwards and futures is that futures are settled daily. As for a detailed discussion of the similarities and differences between forward contracts and futures contracts, interested readers can refer to [Jarrow1981]. Under frictionless markets and continuous trading, simple arbitrage
arguments are invoked to value forward contracts, to relate forward prices and spot prices, and to relate forward prices and futures prices. We also argue that forward prices need not equal futures prices unless default free interest rates are deterministic.

Like forwards, futures are also often used in the electricity markets. Huisman and Kilic analyzed futures prices from the Dutch market, where power is produced with storable fossil fuels, and from the NordPool market where electricity is mostly produced by hydropower, to examine to what extent electricity futures prices contain expected risk premiums or have power to forecast spot prices and whether this might be dependent on the type of electricity supply [Huisman2012]. Lucia and Torró examined empirically the relationship between electricity spot and futures prices by analyzing ten-year data for a set of short term-to-maturity futures contracts traded in the Nordic Power Exchange and found averagely there were significant positive risk premiums in short-term electricity futures prices [Lucia2011].

1.2.1.1.3 Swap

A swap is an agreement between two parties to exchange cash flow in the future. A famous example is a plain vanilla interest rate swap where a fixed rate of interest is exchanged for LIBOR (LIBOR (London Interbank Offered Rate) is the rate at which a bank offers to make large wholesale deposits with another bank). There are different kinds of swaps, for example, credit default swap (CDS). CDS is a specific kind of counterparty financial agreement that provides credit risk protection. It is like an insurance policy. In the event of default or downgrade, the buyer of the CDS receives
payoff from the seller. In general, the seller of the CDS receives payment from the buyer regularly to compensate for providing protection. Therefore, project developers can subscribe a CDS to protect themselves in case of default [Wing2015]. Our focus is energy swap, which is a financial instrument often used by energy utility owner/operators. Since energy utility owner/operators are often thinly capitalized, they are involved into a financial contract with a banker. In this contract, the energy utility owner/operators agree to pay the banker a certain mount of money initially and the banker will take care of the energy cost of the energy utility owner/operators for a certain time period as written in the contract. Swendle [Swendle2014] introduced a few energy swaps, like WTI (West Texas Intermediate) swap, natural gas penultimate swap, and gas daily swaps in his book. However, there are relatively few literature on ‘energy swap’, especially ‘valuation of an energy swap’. This project should be considered frontier study combing chemical process with financial instrument valuation.

1.2.1.1.4 Option

Options are traded both on exchanges and in the OTC market. Most of the options traded on exchanges are American. An option is a contract which gives the buyer (the owner) the right, but not the obligation, to buy or sell an underlying asset or instrument at a specified price on or before a specified date. There are two basic types of options, a call option and a put option. A call option gives the holder the right to buy the underlying asset by a certain date for a certain price while a put option gives the holder the right to sell the underlying assets by a certain date for a certain
price. Suppose there is no premium, the payoffs from buying a call option and a put option are shown in figure 1.1 and 1.2, respectively. The price in the contract is known as exercise price or strike price; the date in the contract is known as expiration date or maturity. American options can be exercised at any time up to the expiration date, but European options can be exercised only on the expiration date or maturity.

![Diagram of Payoff from Buying a Call Option](image)

Figure 1.1: Payoff from buying a call option
A real option technique is used for almost all fields of energy decision making, from electricity generation technologies appraisal to policy evaluation [Fernandes2011]. A real option is an option on real assets (not financial instruments). For example, the opportunity to invest in the expansion of a firm’s factory, or alternatively to sell the factory, is a real call or put option, respectively. Dai et al. constructed a real option valuation model and used this model to valuate renewable energy projects considering selling green electricity certificate revenue. Then they verified the effectiveness and rationality of this model through the investment projects of the wind power [Dai2015]. Boomsma et al. used a real options approach to analyze investment timing and capacity choice for renewable energy projects under different support schemes. A case study based on wind power was given [Boomsma2012]. Fernandes et al. [Fernandes2011] wrote a review paper on the application of real option approach for investments in non-
renewable energy sources and renewable energy sources. Basic principles of real option theory were introduced. Valuation and application of real options were also stated. Moreover, future research directions were pointed out. There are some situations where real options are not proper to use, e.g. when there are no options at all, when there is little uncertainty and when consequences of uncertainty can be ignored. [Schwartz2012]

1.2.1.2 Valuation methods

In this section, three main valuation methods, Monte Carlo method, scenario trees method and PDE method are introduced. In Fernandes’ review paper, a detailed summary of the valuation methods used by researchers from 1987 to 2011 was listed [Fernandes2011].

1.2.1.2.1 Monte Carlo Method

Monte Carlo method is an algorithm that obtains the numerical results with repeated random samplings. It is easy to understand and execute, helpful for problems with uncertainties. For example, to get the probability of obtaining a head by throwing a coin, we can throw the coin for $N$ times and count the number of heads. If $N$ is big enough, we will see the probability to obtain a head is $\frac{1}{2}$.

Monte Carlo method is widely used in energy risk management. For example, Vithayasrichareon and MacGill used Monte Carlo method to develop a novel decision-support tool for assessing future generation portfolios in electricity industry
Shabani and Sowlati combined Monte Carlo Simulation and optimization model to evaluate the impact of uncertainty in biomass quality, availability and cost, and electricity prices on the supply chain of a forest biomass power plant [Shabani2015]. Caporin et al. considered the average and variance interdependence between temperature and energy price series and proposed a Monte Carlo pricing framework for energy and temperature Quanto options [Caporin2012]. However, Monte Carlo method has its limitations. It can’t handle problems with embedded optimization problems, especially when the optimization problem is very complicated. Besides, Monte Carlo method can be computationally expensive.

1.2.1.2.2 Scenario trees

Scenario tree method is to build a tree for an option settlement price that defines the movements, up, down or no change, from node to node of the option settlement price from now until the time of option expiration. The commonly used tree methods include binomial trees and trinomial trees as shown in figure 1.3 and 1.4, respectively.

In scenario tree method, trees can be relatively easy to build and use in option pricing. Also it allows up to incorporating the volatility structure within the tree. However, when there are too many nodes, this method can be time-consuming.
Figure 1.3: Binomial tree with initial price $S_0$
(Price $S$ can go up to $S_u$ or go down to $S_d$ with certain probability at the next time step.)

Figure 1.4: Trinomial tree with initial price $S_0$
(Price $S$ go up to $S_u$ or keep the same or go down to $S_d$ with certain probability at the next time step.)
Portfolio and risk management problems of power utilities can be modeled by multistage stochastic process, which can be simulated by scenarios and corresponding probabilities. As for the computation complexity and time-limitation problem, Growe-Kusta et al. proposed a scenario tree construction algorithm and successfully reduced the nodes by modifying the tree structure and by bundling similar scenarios [Kusta2003]. Eichhorn et al. discussed another big issue of scenario tree modeling in power management, which is appropriate discrete approximation of the underlying stochastic parameters, and suggested to incorporate the so-called polyhedral risk functional into stochastic programs to avoid the risks [Eichhorn2010]. Besides, this method requires no arbitrage and can’t hand price model with spikes or jumps.

1.2.1.2.3 PDE method

As the name implies, PDE method solves problems with a PDE. Sometimes the PDE can be solved with closed-form solution, for example, the famous Black-Scholes equation

$$ \frac{\partial C}{\partial t} + \frac{1}{2} \frac{\partial^2 C}{\partial S^2} \sigma^2 S^2 + r S \frac{\partial C}{\partial S} - r C = 0 , $$

where $C$ is call option price, $S$ is spot price, $K$ is strike price, $r$ is discount rate, and $\sigma$ is volatility. Volatility is an important characteristic of price, which will be introduced in section 1.3.

PDE method is the most popular method recently in option valuation based on the statistical result in [Fernandes2011]. Thompson used PDE method for valuation, optimization and risk management of gas-fired power plants in deregulated markets
Thompson et al. developed a general PDE model for the value of financial derivatives with bilateral counterparty risk and funding costs [Burgard2011]. Hirs also introduced PDE and PIDE method in pricing derivative products and their applications [Hirs2013]. A closed-form solution for a PDE is an ideal implementation method. It is quick and straightforward. However, it is difficult to realize, especially for complicated PDEs. At this time, numerical methods need to be used. Besides, proper boundary conditions are needed. Duffy introduced the application of finite different methods (FDM) in approximating one-factor and multi-factor PDE models for derivative products such as real options [Duffy2013]. Finite difference theory has been used to solve PDE models in physical sciences and engineering for more than 200 years. We will use this method in our research also.

1.2.1.3 Validation

Validation checks the accuracy of a model's representation of a real system. Model validation is defined to mean “substantiation that a computerized model within its domain of applicability possesses a satisfactory range of accuracy consistent with the intended application of the model” [Schlesinger1979]. A model should be built for a specific purpose or set of objectives and its validity determined for that purpose [Sargent2011].

Sargent [Sargent2011] pointed out that there are four basic decision-making approaches for deciding whether a simulation model is valid or not. One approach is for the model development team itself to make the decision about the validity of a
A subjective decision is made based on the results of the various tests and evaluations conducted as part of the model development process. Another approach is to involve the user(s) of the model heavily with the model development team in deciding whether the simulation model is valid if the size of the simulation team developing the model is small. In this approach the focus of determining the validity of the simulation model transfers from the model developers to the model users. Also, this approach helps increase model credibility. The third one is called ‘independent verification and validation’ (IV&V), using a third (independent) party to decide whether the simulation model is valid. The IV&V approach should be used when developing large-scale simulation models, whose developments usually involve a few teams. It is usually better to use the second and the third approaches for determining model validity than the first one. The last approach is to use a scoring model (see [Chen2012], [Chi2012] for examples of scoring models). Scores (or weights) are determined subjectively when conducting various aspects of the validation process and then combined to determine category scores and an overall score for the simulation model. A simulation model is considered valid if its overall and category scores are greater than some passing score(s). This approach is seldom used in practice. Besides, he also talked about validation techniques including comparison of other models, historical data validation, which are used in this dissertation. Similar validation methods are also executed in [Gökgöz2012]. Gökgöz and Atmaca used Turkish historical balanced market hourly system marginal and day-ahead hourly market prices between of 2006 and 2011 for portfolio optimization. Besides, Lee et al. did a case study with real data in a steel
making company in India to determine the optimal procurement quantity for profit maximization [Lee2014]. Thompson gave a realistic example for natural gas storage valuation and optimization with real option technique [Thompson2009].

1.2.2 Physical storage

In this section, storage facility type for oil, coal, and natural gas will be introduced. Methods of obtaining optimal control operation will be also described.

1.2.2.1 Storage facility type

1.2.2.1.1 Oil storage

Oil is often stored in storage tanks, which comes in all sizes and shapes. Special applications might require tanks to be rectangular, in the form of horizontal cylinders, or even spherical in shape. Which type of storage tank will be used may depend on the nature of the stored product, pressure, environmental regulation, etc.

1.2.2.1.2 Coal storage

Coal is often stored on clay or sandy ground. The coal storage site must be properly chosen and prepared. The site should be dry, level, well drained. Also it should be cleared of any vegetation and make sure the site is away from any external heat sources as combustion liability increases with a rise in temperature.
1.2.2.1.3 Natural gas storage

There are three main types of natural gas storage facilities widely used in US and Canada, which include depleted oil and gas reservoirs, salt cavern storage, and aquifer storage. They all require certain amount of cushion/base gas, where aquifer reservoirs require the most, 50%-80%; depleted reservoirs require 50%; and salt cavern storage facilities require 20-30%. Other important characters for natural gas reservoirs include working gas capacity, deliverability, injection capacity and cycling. Working gas capacity is the amount of gas that is available to produce and sell. Deliverability means how fast a reservoir can release gas while injection capacity means how fast gas can be pumped into a reservoir. Deliverability and injection capacity depend on the pressure in the reservoir. Cycling means how often the working gas volumes can be injected and withdrawn in a year. Although a depleted gas reservoir has relatively high base gas capacity, it is the most commonly form of underground storage since it is generally the cheapest and easiest storage facility to develop, operate, and maintain. [Thompson2009]

1.2.2.2 Methods of obtaining optimal storage

Control strategy is to tell facility owner/operators when to buy or sell energy commodities. In principle, the idea is 'buy low, sell high', which means when energy commodity price is relatively low, energy facility owner/operators buy commodity and store while when its price is relatively high, they sell to make profits. In this section,
three control methods, including threshold method, receding horizon control method and PED method are introduced.

1.2.2.2.1 Threshold method

In this method, threshold values are given to obtain the control strategy. For example, for crude oil storage, if the spot price $S(t) \leq S_{ths}$, crude oil is injected into the storage tank, while if $S(t) \geq S_{thr}$, crude oil is released from the storage tank, where $S_{ths}$ and $S_{thr}$ are the threshold values for storage and releasing, respectively. Mattingley et al. [Mattingley2011] compared this method with receding horizon policy (see section 1.2.2.2.2) and the results showed the receding horizon policy is superior to the threshold method. This method can be used empirically but it is difficult to obtain the threshold values.

1.2.2.2.2 Receding horizon control method

Receding horizon control (RHC), also known as model predictive control (MPC) [Rawlings2009], is a feedback control technique that became popular in the 1980s. It is a nonlinear control policy, often superior to linear control [Rawlings2009b]. RHC can be used in a lot of fields involving control, like industrial and chemical process control [QinBadgwell2003]. Mattingley et al. [Mattingley2011] gave a few examples of its application, like preordering and processor speed control. Although RHC is of great applications, it doesn't always outperform traditional control methods, e.g. PID (proportional-integral-derivative) control. Besides, a RHC optimization problem must be solved at each time step.
1.2.2.3 PDE method

PDE method is also named dynamic programming method. In this method, a PDE including control variables needs to be solved. By solving the PDE, the optimal control is meanwhile obtained since the control optimization problem is embedded. This method is also used for valuation as introduced in section 1.2.1.2.3. We employ the PDE method in this dissertation. A PDE, named Hamilton-Jacobi-Bellman (HJB) equation is derived in section 2.3.1. A similar derivation process can also be found in [Merton1976], [Merton1990], [Thompson2004], and [Thompson2009]. Xu [Xu2004] also obtained a PDE by hedging. Given proper boundary conditions, a PDE can be solved analytically or numerically. PDE method can be used for transient systems. It is said a 'natural' approach to solve valuation and optimal operation problem [Lai2008].

1.3 Energy commodity price modeling

Energy commodity price modeling is important for obtaining energy risk management strategies since risks come from price fluctuation. There are different prices and price characteristics. Which price and/or price characteristics are used in price model may lead to different optimal operation strategies for energy commodity storage. Also, a price model including as many characteristics as possible can capturer price trends more accurately but meanwhile make the price model more complicated. In this section, three primary types of prices are first introduced, followed by price characteristics. Then various price models based on the number of factors are described in detail.
1.3.1 Price

There are three primary types of prices in the commodity market, spot price, forward price, and future price.

- Spot price – it is the price that is quoted for immediate settlement (payment and delivery).
- Forward price – it is the agreed upon price of an asset in a forward contract.
- Future price – it is the price agreed in a future contract.

In energy commodity markets, forward price and future price can be used interchangeably [Pilipovic1998]. The relationship between spot price $S$ and future price $f(T,t)$ is

$$f(T,t) = S(t)e^{(r-y)(T-t)},$$  \hspace{1cm} (2.1)

where $f(T,t)$ represents future price at maturity $T$; $S(t)$ represents spot price at time $t$; $r$ is risk-free interest rate; $y$ is convenience yield. A convenience yield is an adjustment to the cost of carry in the non-arbitrage pricing formula for forward prices in markets with trading constraints.

1.3.2 Price characteristics

1.3.2.1 Volatility

Volatility is an important characteristic of energy commodity price. Spot price volatility tells us about how much randomness there is in the spot price return over very small time intervals. Volatility isn’t a constant. It can be very volatile. In fact, volatility in
most energy markets is a function of time, exhibiting a combination of deterministic and random behavior that can exhibit different characteristics in the short term when compared with the long term. To simply price modeling, many people treat it as a constant.

1.3.2.2 Seasonality

Seasonality is another important feature for commodity prices affected by weather change. For example, North American natural gas markets exhibit seasonal variation with higher prices in winter because of increased heating demand. This seasonality of prices can be seen in natural gas future contracts traded on the New York Mercantile Exchange (Nymex). Several factors contribute to the reduction in seasonality in natural gas markets: increase natural gas production, increase natural gas storage capacity, use other energy commodity like coal instead of natural gas. The market for future year contracts currently reflects lower seasonal variability than in the past as reported by Energy Information Administration (EIA).

1.3.2.3 Mean-reversion

Mean-Reversion is the tendency of a stochastic process to return over time to a long-run average value. Due to unbalanced supply and demand, energy commodity prices goes up and down and finally present mean-reversion. Mean-reversion is a natural instinct of energy price. When supply is greater than demand, energy commodity price goes down, suppliers see little profit and will stop selling, and supply
and demand tends to reach the balance. If a lot of suppliers drop their supplies together, supply may be less than demand, and energy commodity price will go up. At that time, more supplies will swarm into the market. Therefore the market is self-adjusted based on the dynamic relationship between supply and demand and make commodity prices mean-reverting. Because of this characteristic, the investors would like to take potential risks for benefits since there will be one day when energy commodity price will go up or drop down. Or else it doesn’t make any sense to make any investment since everything is just random.

1.3.2.4 Jumps and spikes

Jumps or spikes are the abrupt and unanticipated extreme changes in the spot prices. Within a very short period of time, the system price can increase substantially and then drop back to the previous level. Jumps in the spot prices are an effect of extreme load fluctuations, caused by severe weather conditions often in combination with generation outages or transmission failures. These spikes are normally quite short lived, and as soon as the weather phenomenon or outage is over, prices fall back to a normal level [Deng2000].

1.3.3 Price model

Price models of commodities can be divided into one-factor\(^1\) models, two-factor models, and multi-factor models. These models can include one (for one factor model)

\(^1\) A factor represents a market variable exhibiting some form of random behavior.
or more (for two- and multi-factor models) of the characteristics of prices, e.g. volatility, mean-reversion, seasonality, jumps and spikes as introduced in section 1.3.2, and/or other factor(s) a price may exhibit, for example, convenience yield.

1.3.3.1 One-factor model

A one-factor model is a model containing only one independent random variable. Xu [Xu2004] introduced a few one-factor price models for natural gas, including a general one (see equation (2.2)), one without seasonality (see equation (2.3)), and one with seasonality (see equation (2.4)-(2.7)),

$$dS = \alpha(t,S)dt + \sigma(t,S)dZ,$$  \hspace{1cm} (2.2)

where $Z$ is a Wiener process, $\alpha(t,S)$ and $\sigma(t,S)$ are deterministic functions of $t$ and $S$.

$$dS = \alpha(L-S)dt + \sigma^rS^r dZ,$$ \hspace{1cm} (2.3)

where $\alpha$ is the mean reversion rate, $L$ is the long term mean and $\sigma$ is the diffusion term. They are all constants. $r=0$ or $\frac{1}{2}$. When $r=0$, the model becomes an Ornstein-Uhlenbeck process, which is also used in [Schwartz1997]; when $r=\frac{1}{2}$, it is called Cox-Ingersoll-Ross model. Equation (2.3) can be simply obtained by substituting $\alpha(t,S)=\alpha(L-S)$, $\sigma(t,S)=\sigma^r$ in equation (2.2). Equation (2.4) is composed of two parts, one representing the seasonality $f(t)$, the other representing unseasonal term and following a one-factor mean-reverting process $X$,

$$S = f(t) + X,$$ \hspace{1cm} (2.4)

$$f(t) = bt + \sum_{i=1}^{N} \beta_i \cos(\frac{2\pi it}{P}) + \eta_i \sin(\frac{2\pi it}{P}),$$ \hspace{1cm} (2.5)
\[ dX = \alpha(L - X)dt + \sigma(t)X' dZ, \]  
\[ \sigma(t) = \exp(c + \sum_{j=1}^{n} \lambda_j \cos\left(\frac{2\pi j t}{P}\right) + \omega_j \sin\left(\frac{2\pi j t}{P}\right)). \]

In equation (2.4)-(2.7), \( b, \beta, \eta, \alpha, L, c, \lambda, \omega \) are constants. \( N \) and \( K \) are positive integers. \( Z \) is a Wiener process. \( r = 0, 1 \) or \( \frac{1}{2} \). \( P \) is the number of trading days in one year (251 days usually).

Chen and Forsyth [Chen2007] contained the seasonality effect into a price model similar to equation (2.3), but with \( L \) as a function of \( t \)
\[ L(t) = L_0 + \beta_{SA} \sin\left(4\pi(t - t_{SA})\right), \]

where \( L_0 \) is the equilibrium price without seasonality effect; \( \beta_{SA} \) is the semiannual seasonality parameter; \( t_{SA} \) is the seasonality centering parameters, representing the time of semiannual peak of equilibrium price in summer and winter. Chen and Forsyth [Chen2007b] also developed two one-factor regime-switching models for natural gas spot price, a mean-reverting model and a Geometric Brownian Motion (GBM) model, which could exhibit a behavior similar to the two-factor model in [Xu2004] and in [Schwartz1997], respectively. The prices could switch between one regime with a lower equilibrium price and the other regime with a higher equilibrium price. The switch between two regimes was modeled by a two-state continuous-time Markov chain. The regime-switching GBM model could capture both the short-term and long-term dynamics of market futures prices while the pure mean-reverting and the regime-switching mean-reverting models could not. However, by using the regime-switching GBM model to price the value of a natural gas storage facility, the obtained optimal
control was independent of the gas price (unless at P=0), which is controversial comparing with the normal optimal strategy 'buy low, sell high'.

Spot price jumps are common to see with a high demand. Deng [Deng2000] proposed several mean-reversion jump diffusion models to describe energy commodity spot prices. Thompson etc. [Thompson2009] used the most general form of a continuous time Markov process including jumps for the price of natural gas

\[ dS = \mu(S,t)dt + \sigma(S,t)dZ + \sum_{k=1}^{N} \gamma_k(S,t,J_k)dq_k, \]

where \( \mu, \sigma, \) and \( \gamma_k \)s can be any arbitrary functions of price and/or time, and the \( J_k \)s are drawn from some other arbitrary distributions \( Q_k(l) \). \( dq_k \)s are Poisson processes with the properties

\[ dq_k = \begin{cases} 
0 \text{ with probability } 1 - \varepsilon_k(S,t)dt \\
1 \text{ with probability } \varepsilon_k(S,t)dt 
\end{cases}, \]

where \( \varepsilon_k(S,t) \) is jump intensity, representing the mean arrival time of the Poisson process. Chen and Forsyth [Chen2007] also considered jump effects for spot gas prices. They included a jump term in a pure mean-reverting process. Instead of treating the jump intensity a function of price and/or time, they treated it as a constant.

1.3.3.2 Two-factor model

Similar to one-factor model, a two-factor model contains two random but possibly correlated variables.
R. Gibson and E. Schwartz [GibsonSchwartz1990] developed a two-factor model of natural gas. The two random variables are spot price $S$ and convenience yield $y$. The formulae are

\[ dS = \mu Sdt + \sigma_1 SdZ_1 , \]
\[ dy = k(\alpha - y)dt + \sigma_2 dZ_2 , \]
\[ dZ_1 dZ_2 = \rho dt , \]

where $\mu$, $\sigma_1$, $\sigma_2$, $k$ and $\alpha$ are parameters. $Z_1$ and $Z_2$ are Wiener processes. The increments of the two Wiener process $dZ_1$ and $dZ_2$ are correlated with correlation coefficient $\rho$. This model works well for short-term financial instruments, such as future contracts, but can't capture long-term dynamics of market prices.

1.3.3.3 Multi-factor model

A multi-factor model is a price model contains more than two random variables. Schwartz [Schwartz1997] used a three-factor model for pricing future contracts. Not only did they treat $S$ and $y$ stochastically, but also the interest rate $r$. The specific expressions are

\[ dS = (r - y)Sdt + \sigma_1 SdZ_1 , \]
\[ dy = k(\alpha - y)dt + \sigma_2 dZ_2 , \]
\[ dr = a(m - r)dt + \sigma_3 dZ_3 , \]
\[ dZ_1 dZ_2 = \rho_1 dt , \]
\[ dZ_2 dZ_3 = \rho_2 dt , \]
\[ dZ_1 dZ_3 = \rho_3 dt , \]
\[ dZ_i dZ_j = \rho_{i,j} dt, \]

where \( \sigma_i \) (i=1,2,3) is the standard deviation; a and k are the speed of adjustment coefficients; \( m \) is the risk adjusted mean short rate of the interest rate process. \( Z_i \) (i=1,2,3) is a Wiener process, correlated with each other with the correlation coefficient \( \rho (i=1,2,3) \), respectively. Lai etc. [Lai2008] used an N-factor model as a representative of the high-dimensional forward models, which seems to be used in practice:

\[
\frac{dF(t,T_i)}{F(t,T_i)} = \sigma_i dZ_i(t), \quad \forall i \in \mathcal{F}
\]

\[
dZ_i(t)dZ_j(t) = \rho_{i,j} dt, \quad \forall i \in \mathcal{F}
\]

where \( \mathcal{F} = \{0,\ldots,N-1\} \). \( F(T_i, T_j) \) denotes the futures price at time \( T_i \) with maturity at time \( T_j \), \( \forall i, j \in \mathcal{F}, j \geq i \). \( F(T_i, T_j) \) is the spot price at time \( T_i \). In this model, the risk-neutral dynamics of the price of the natural gas futures, which mature at \( T_i \), are described by a driftless GBM with maturity-specific constant volatility \( \sigma_i \) and standard Brownian motion increment \( dZ_i(t) \), where \( dZ_i(t) \) and \( dZ_j(t) \) are correlated with the constant correlation coefficient \( \rho_{i,j} \in (-1,1) \). When \( i=j \), \( \rho_{i,i} = 1 \).

Multi-factor models predict prices more accurately but they are more complicated and make computation more expensive.
CHAPTER 2:

VALUATION OF AN ENERGY SWAP FOR A FLEXIBLE FUEL PROCESS

2.1 Introduction

Chemical commodities and energy operations are primary elements of the global economy. Participants in the value chain of these industries are subject to financial risks due to volatility of the commodity markets, uncertain demand, and future events outside the participants' direct control. Uncontrolled risks may take a financially unsustainable toll on these participants. Risk assessment and mitigation are major functions of the commodity markets [Geman2005; Geman2008]. Real options technique is able to obtain the optimal operating rule and explicitly capture the flexibility and its effect on uncertainty and provide a consistent treatment of risk in the valuation of flexible projects. [Kulatilaka1993]

A real option is “a right, but not the obligation, to take an action (e.g., deferring, expanding, contracting or abandoning) at a predetermined cost, called exercise price, for a predetermined period of time – the life of the option.” [Copeland2003]. Coined in 1977 by Stewart Myers, it has been used in many areas including marine engineering [Knight2014], energy investment [Lee2011, Blanco2011], and automotive industry [Avadikyan2010]. Flexible operations, either flexible production or flexible working
modes, have been analyzed using an option by quite a few researchers. Kulatilaka [Kulatilaka1993] evaluated the case of a dual-fuel industrial steam boiler and showed the dual-fuel steam boiler, which could use either natural gas or oil, was less costly than operate with a rigid, single-fuel alternative. However, the correlation of the two energy commodities was not considered. Triantis and Hodder [Triantis1990] developed an approach for valuing flexible production systems using contingent claims pricing and illustrated their approach by valuing a flexible system that produced two products. Trigeorgis [Trigeorgis1993] wrote a review paper about real options and their interaction with financial flexibility, which described the evolution of real options and how to quantify in principle the value of various types of operating options embedded in capital investment. Smit et al. [Smit2006] also used examples and cases to illustrate the use of real options valuation and game theory principles to analyze prototypical investment opportunities involving important competitive/strategic decisions under uncertainty.

There is a significant literature about real options on energy risk management, but relatively few reports on ‘energy swaps’. An energy swap is a contract between energy utility owner/operators and a banker, where energy owner/operators pay the banker a certain amount of money initially and the banker will be responsible for their energy cost for a certain period of time they agreed on. Since energy utility owner/operators are often thinly capitalized and easily subject to the operational risks caused by energy price fluctuations even with flexible fuel processes, they are often involved into an energy swap. A key issue here is to decide a ‘fair value’ for them to
accept the contract. The determination of the 'fair value' provides a standard measure for the risk-adjusted financial return. A flexible process for which financial return and financial risk are quantified can be compared to alternative instrument opportunities with known value. This approach of valuation has been widely adapted in the financial services industry [Biger1983, Hull1990] and for management of tangible assets in the 'real options' literature [Trigeorgis1996, Trigeorgis1993, Luenberger1998, Shockley2006]. We adopt this strategy in an ‘energy swap’ in order to assign value to process flexibility, in particular the capability to use flexible fuels in an energy utility.

Values of flexible process operations can be obtained by the integration of operations decisions, which can be a partner with financial operation. Optimal coordination of process and financial operations provide boundaries for process valuation. In the real options approach to process valuation [Schwartz2001], the financial performance of a process is approximated as a financial option constructed using the commodity markets. For example, valuation of natural gas storage facilities provides an interesting case study shown in the work by Chen and Forsyth [Chen2007], Thompson et al. [Thompson2009], and Secomandi et al. [Secomandi2010, Lai2010]. Financial returns can be exactly replicated using stochastic price models of the underlying commodities. The well-known option pricing models can be used to compute a unique process value. In the case of 'incomplete markets', however, process returns cannot be exactly projected onto the underlying stochastic price models. In these cases, process valuation is not unique and depends on operational decision.
In our work, the operational integration of a flexible fuel (natural gas and coal) system, as an example of simple flexible energy operation processes, is evaluated by means of the widely used financial instrument energy swap. In the valuation process, a hedging portfolio is constructed including a flexible fuel process, the investment on risk-free assets and the physical ownership of the energy commodities. The price correlation of the two energy commodities is considered. The purpose of this work is to propose a framework based on steady-state mass and energy balances to analyze strategies for mitigating financial risks for a simple model of an energy utility. This modeling framework is similar in philosophy to those used for process targeting in [Patel2007, Glasser2009, Hildebrandt2009]. Our working hypothesis is that integrating process and financial operations of commodity processes provides additional opportunities for enhancing returns and mitigating financial risk.

This chapter is organized as follows. In section 2.2, stochastic model of flexible fuel processes is introduced. Valuation of an energy swap is developed in section 2.3. PDE method is used and a PDE is obtained with the control variable embedded into a linear optimization problem. Parameters in stochastic correlated energy commodity price models are fitted and a finite difference method is used to solve the PDE to obtain the optimal control/hedging. In section 2.4, validation computation is executed. Simulated price data with Monte Carlo method and historical price data are both used to validate the valuation results.
2.2 Stochastic model of flexible fuel process

2.2.1 Simple process

As stated in [Garcia2012], the stoichiometry of a simple process is formulated in terms of the raw materials and final products as shown in equation (2.1) [Feinberg1974]. Following standard conventions for process stoichiometry, \( \varepsilon = [\varepsilon_{e,s}] \) is the molecular matrix where \( \varepsilon_{e,s} \) denotes the number of atoms of element \( e \) (\( e=1,2,\ldots,E \)) in species \( s \) (\( s=1,2,\ldots,S \)). Matrix \( v=[v_{r,s}] \) is the matrix of stoichiometric coefficients where subscript \( r \) denotes the reaction (\( r=1,2,\ldots,R \)). Stoichiometric coefficients are in the null space of the molecular matrix.

\[
\sum_{s=1}^{S} \varepsilon_{e,s} v_{s,r} = 0, \ \forall e, \forall r , \tag{2.1}
\]

A simple process takes a set of reactions and combines them in a certain manner to achieve a desired net final stoichiometry, \( v_{s,\text{net}} \). The decision variable in a simple process then is one that quantifies how chemical reactants are transformed into products, known as \( \chi_{r} \) or `extent of reaction'.

In order to calculate a net profit from process operation, we introduce constants that specify pricing information. \( C_{s} \) represents the costs of species present in all reactions and \( Q_{r} \) corresponds to the intrinsic cost associated to running a particular reaction in the process. Matrices \( A_{c,s} \), \( B_{c,r} \) and \( C_{c} \) are defined to describe capacity and other linear constraints on the process. Given this information, a simple process is one that maximizes profit according to
\[
\text{Profit} = \max_{\chi_r} \left[ \sum_{s=1}^{S} C_s v_{s}^{\text{net}} - \sum_{r=1}^{R} Q_r \chi_r \right]
\]

subject to

\[
v_{s}^{\text{net}} = \sum_{r=1}^{R} v_{s,r} \chi_r ,
\]

\[
\sum_{s=1}^{S} A_{c,s} v_{s}^{\text{net}} + \sum_{r=1}^{R} B_{c,r} \chi_r \geq C_c,
\]

which is illustrated in figure 2.1 [Garcia2012].

![Simple process model](image)

**Figure 2.1: Simple process**

### 2.2.2 Flexible fuel model

Simple process models include a wide range of examples encountered in the market for energy derivatives, power plant operations, commodity chemicals, and other commodity conversion operations, for example, flexible fuel utilities. In flexible fuel processes operators can choose between a combination of fuels to fulfill energy demands. This combination is chosen so as to meet capacity and ensure that cost is minimized. For example, a utility can convert natural gas (NG) or coal (QL) or the combination of them into electricity [Kantor2012], as illustrated in figure 2.2.
The task is to find the optimal mixture of fuels in order to minimize the cost. The optimization problem can be formed as

\[ C(S_{NG}, S_{QL}) = \min_q (S_{NG}H_{NG}q_{NG} + S_{QL}H_{QL}q_{QL}), \]

subject to

\[ q_{NG} + q_{QL} = 1 \]
\[ q_{NG} \geq 0, q_{QL} \geq 0 \]

where \( C(S_{NG}, S_{QL}) \) is the minimum cost function which depends on the stochastic prices of natural gas and coal, \( S_{NG} \) and \( S_{QL} \); \( H_{NG} \) and \( H_{QL} \) are constant heating rates of natural gas and coal, respectively; \( q_{NG} \) and \( q_{NG} \) are heat input ratios of natural gas and coal, respectively.

2.3 Problem formulation

2.3.1 Model development

The volatile energy markets expose utility providers to significant price uncertainty. Figure 2.3, for example, shows the price of the near future contract (an
approximation to the `spot' price) for natural gas and Appalachian coal. We consider a utility operator with the operational flexibility to use either natural gas or coal, in any combination, to meet a fixed demand.

Utility operators are often thinly capitalized and work in tightly regulated retail markets. Therefore the operator will enter into an `energy swap' with a banker who, for a fixed payment, underwrites the utility's cost of fuel. A swap is a market derivative in which one counter-party pays another a fixed payment and in exchange receives a payment linked to a certain floating index [Eydeland2003]. In this energy swap, the utility operators provide a fixed payment $V(S_{NG}(t_0), S_{QL}(t_0), t_0)$ at $t_0$, where $S_{NG}(t_0)$ and $S_{QL}(t_0)$ are the spot market prices of natural gas and coal at $t_0$, respectively. In return, the banker pays the utility fuel cost $C(S_{NG}(t), S_{QL}(t))$ during an agreed time period. This simple
type of energy swap is illustrated in figure 2.4. The problem is to determine a ‘fair’ price for the utility owner/operators to pay the energy banker for the obligation to finance the utility's energy costs.

\[
\begin{align*}
\text{Fixed Payment} \\
V(S_{NG}(t_0), S_{QL}(t_0), t_0) \\
\text{Utility} & \quad \rightarrow \quad \text{Banker} \\
\text{Uncertain Fuel Cost} \\
C(S_{NG}(t), S_{QL}(t))
\end{align*}
\]

Figure 2.4: Illustration of a simple energy swap

Drawing on the literature for financial derivatives, the standard approach to this problem is to determine the initial capital needs for the energy banker to set up and operate a risk-free hedge [Hull2009; Hull2012]. The hedge consists of a flexible fuel process, a portfolio of cash invested in risk-free assets and the ownership of natural gas and coal inventories. The portfolio is called a self-financing hedging portfolio since there is no exogenous infusion or withdrawal of money. The banker extracts money from the portfolio to cover the utility's fuel costs. Given stochastic models for the evolution of energy prices, the stochastic control problem is to manage the portfolio to pay the fuel costs of utility while minimizing or eliminating risks. If risks can be completely eliminated then the no-arbitrage price of the energy swap is the value of the capital required to set up the hedge.
The self-financing hedging portfolio \( V \) is composed of cash, physical ownership of \( \theta_{NG} \) units of natural gas, \( \theta_{QL} \) units of coal, and the fuel-financing obligation. The value of the portfolio is a stochastic process as shown in equation (2.2),

\[
dV = r(V - \theta_{NG}S_{NG} - \theta_{QL}S_{QL})dt + \theta_{NG}dS_{NG} + \theta_{QL}dS_{QL} - C(S_{NG}, S_{QL})dt + \theta_{NG}y_{NG}dt + \theta_{QL}y_{QL}dt,
\]

where \( r \) represents the risk-free interest rate; \( y_{NG} \) and \( y_{QL} \) represent convenience yields for natural gas and coal, respectively. Convenience yield is the net return to the portfolio attributable to physical ownership of the underlying commodities. It is a common feature of commodity price models but not commonly shared by other financial instruments. Convenience yield is positive if the owner places a high intrinsic value on physical ownership, such as the avoidance of plant shutdowns in the event of raw material shortages. It may be negative if the cost-of-carry of the physical inventory is large.

Natural gas and coal prices are modeled as general Itô processes as shown in equation (2.3) and (2.4),

\[
dS_{NG} = a_{NG}(S_{NG})dt + \sigma_{NG}S_{NG}dZ_{NG}, \tag{2.3}
\]

\[
dS_{QL} = a_{QL}(S_{QL})dt + \sigma_{QL}S_{QL}dZ_{QL}, \tag{2.4}
\]

where \( a(S) \) is called drift; \( \sigma \) is volatility; \( dZ_{NG} \) and \( dZ_{QL} \) are correlated independent identically distributed stochastic processes. For commodities, the deterministic portion of the returns, \( a(S) \), typically exhibits mean-reversion.
A necessary input to this model is the cost of fuel required to meet the utilities production requirement. Process flexibility provides the operator with the ability to respond to market price changes. The banker assumes the operator will respond by implementing a minimum cost strategy. Our model corresponds to a simple process with flexible fuels [Kantor2012; Mousaw2011; Mousaw2010]. Following this framework, given prices $S_{NG}(t), S_{QL}(t)$ at time $t$, the solution to the minimum fuel cost function $C(S_{NG}(t), S_{QL}(t))$ is found as

$$C(S_{NG}, S_{QL}) = \min(S_{NG}H_{NG}, S_{QL}H_{QL}),$$

which is obtained by choosing the fuel alternatively.

The control task is to manage the hedging portfolio in order to minimize risk. Substituting the price model into the expression for $dV$, using Itô's lemma, and choosing

$$\theta_{NG} = \frac{\partial V}{\partial S_{NG}}$$

and

$$\theta_{QL} = \frac{\partial V}{\partial S_{QL}}$$

produces a risk-free portfolio. (This is a standard technique in the finance literature, for details refer to [Geman2005].) An HJB equation is obtained as shown in equation (2.6). The functions $\theta_{NG}(S_{NG}(t), S_{QL}(t), t)$ and $\theta_{QL}(S_{NG}(t), S_{QL}(t), t)$ implement the optimal hedging as an open loop control policy measuring current spot prices.

$$-\frac{\partial V}{\partial t} = \frac{\sigma_{NG}^2 S_{NG}^2}{2} \frac{\partial^2 V}{\partial S_{NG}^2} + \rho\sigma_{NG} \sigma_{QL} S_{NG} S_{QL} \frac{\partial^2 V}{\partial S_{NG} \partial S_{QL}} + \frac{\sigma_{QL}^2 S_{QL}^2}{2} \frac{\partial^2 V}{\partial S_{QL}^2} + (rS_{NG} - y_{NG}) \frac{\partial V}{\partial S_{NG}} + (rS_{QL} - y_{QL}) \frac{\partial V}{\partial S_{QL}} - rV + C(S_{NG}, S_{QL})$$

If we treat $t'=T-t$, equation (2.6) becomes
\[
\frac{\partial V}{\partial t} = \frac{\sigma_{NG}^2 S_{NG}^2}{2} \frac{\partial^2 V}{\partial S_{NG}^2} + \rho \sigma_{NG} \sigma_{QL} S_{NG} S_{QL} \frac{\partial^2 V}{\partial S_{NG} \partial S_{QL}} + \frac{\sigma_{QL}^2 S_{QL}^2}{2} \frac{\partial^2 V}{\partial S_{QL}^2} + \frac{\sigma_{NG}^2 S_{NG}^2}{2} \frac{\partial^2 V}{\partial S_{NG}^2} + (r S_{NG} - y_{NG}) \frac{\partial V}{\partial S_{NG}} + (r S_{QL} - y_{QL}) \frac{\partial V}{\partial S_{QL}} - r V + C(S_{NG}, S_{QL}).
\]  
(2.7)

2.3.2 Parameter fitting

In equation (2.7), \( \sigma_{NG} \), \( \sigma_{QL} \), and \( \rho \) come from energy price models, which need to be calibrated using historical price information. A mean-reverting model is used to model the price movements for natural gas and coal, respectively in this application given in equation (2.8) - (2.10):

\[
\frac{dS_{NG}}{S_{NG}} = \eta_{NG} (\mu_{NG} - \ln S_{NG}) dt + \sigma_{NG} dZ_{NG},
\]  
(2.8)

\[
\frac{dS_{QL}}{S_{QL}} = \eta_{QL} (\mu_{QL} - \ln S_{QL}) dt + \sigma_{QL} dZ_{QL},
\]  
(2.9)

\[
dZ_{NG} dZ_{QL} = \rho dt.
\]  
(2.10)

Represent \( X = \log(S) \) and apply Itô's lemma, the mean-reverting price model becomes AR(1) model:

\[
dX = \left[ \eta (\mu - X) - \frac{1}{2} \sigma^2 \right] dt + \sigma dZ.
\]  
(2.11)

Details about fitting the price model parameters \( \eta \), \( \mu \), \( \sigma \), and \( \rho \) can be found in [Mousaw2011]. In this subsection, coal and natural gas historical price data (01/01/2010-03/27/2012) are used. The price data are downloaded from the US EIA website. The fitted parameters are listed in table 2.1 and the other parameters of process information are in table 2.2.
TABLE 2.1  
PARAMETERS IN PRICE MODELS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>0.2467</td>
<td>--</td>
<td>Correlation factor</td>
</tr>
<tr>
<td>$\sigma_{NG}$</td>
<td>0.0220</td>
<td>--</td>
<td>Standard deviation for natural gas</td>
</tr>
<tr>
<td>$\sigma_{QL}$</td>
<td>0.0121</td>
<td>--</td>
<td>Standard deviation for coal</td>
</tr>
</tbody>
</table>

TABLE 2.2  
PARAMETERS FOR PROCESS INFORMATION

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r$</td>
<td>$2.6116 \times 10^{-14}$</td>
<td>--</td>
<td>Daily risk-free discount rate</td>
</tr>
<tr>
<td>$H_{NG}$</td>
<td>196.8</td>
<td>MMBtu/MW/d</td>
<td>Heating rate of natural gas</td>
</tr>
<tr>
<td>$H_{QL}$</td>
<td>10.3</td>
<td>ton/MW/d</td>
<td>Heating rate of coal</td>
</tr>
<tr>
<td>$y_{NG}$</td>
<td>0</td>
<td>$$/MMBtu</td>
<td>Convenience yield of natural gas</td>
</tr>
<tr>
<td>$y_{QL}$</td>
<td>0.2</td>
<td>$$/ton</td>
<td>Convenience yield of coal</td>
</tr>
</tbody>
</table>

2.3.3 Initial and boundary conditions

To solve equation (2.7), initial and boundary conditions are needed.

The initial condition in $t$ is $V=0$ when $t=T$ ($t'=0$). That's because when the portfolio matures, it will be worth nothing. Therefore $V=0$.

Since $V$ depends on the second derivative with respect to (w.r.t.) $S_{NG}$ and $S_{QL}$, two boundary conditions are needed for $S_{NG}$ and $S_{QL}$ respectively. Thompson et al. [Thompson2009] pointed out that boundary conditions depended on the specific spot price model and in most cases the boundary conditions below were sufficient.

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These two boundary conditions mean when the price of one kind of energy commodity is very large/small, the utility operators always will not/will use that kind of commodity as the fuel and the hedging strategy will not change. Therefore, in those regions there are no curvatures for $S_{NG}$ and $S_{QL}$.

2.3.4 Numerical schemes

A first-order accurate finite difference method is used for the discretization in $t'$, $S_{NG}$ and $S_{QL}$.

Let $V_{i,j}^k$ represents the value of $V$ at the $i^{th}$ grid point of $S_{NG}$, the $j^{th}$ grid point value of $S_{QL}$ at time step $k$, where $i=1, 2, ..., I-1, I; j=1, 2, ..., J-1, J$. $I$ and $J$ are the total numbers of grid points in $S_{NG}$ and $S_{QL}$, respectively.

In $t'$, the forward discretization formula is used

$$\frac{\partial V_{i,j}^k}{\partial t'} = \frac{V_{i,j}^{k+1} - V_{i,j}^k}{\delta t'}.$$  

Upwind method is used to discretize the first derivatives in $S_{NG}$ and $S_{QL}$, which depend on the sign of the coefficients in front of them. For example, for the first derivative of $S_{NG}$, if $(r_{NG} - y_{NG}) \geq 0$,

$$\frac{\partial V_{i,j}^k}{\partial S_{NG}} = \frac{V_{i+1,j}^k - V_{i,j}^k}{\delta S_{NG}},$$

and if $(r_{NG} - y_{NG}) < 0$, 

$$V_{SS} = 0 \quad \text{for } S \text{ large},$$

$$V_{SS} = 0 \quad \text{as } S \to 0.$$
where $\delta S_{NG}$ is the step size in $S_{NG}$ direction.

Central approximation is used to discretize the second derivatives. For example, the discretization formula for the interior points of the second partial derivative w.r.t. $S_{NG}$ is

$$
\frac{\partial^2 V_{i,j}^k}{\partial S_{NG}^2} = \frac{V_{i+1,j}^k - 2V_{i,j}^k + V_{i-1,j}^k}{(\delta S_{NG})^2}.
$$

Discretization formulae on the boundaries can be obtained by making good use of the boundary conditions, which are

$$
\frac{\partial^2 V_{1,j}^k}{\partial S_{NG}^2} = 0,
$$

$$
\frac{\partial^2 V_{L,j}^k}{\partial S_{NG}^2} = 0.
$$

Discretization formulae are similar for $\frac{\partial V}{\partial S_{QL}}$, $\frac{\partial^2 V}{\partial S_{QL}^2}$, and $\frac{\partial^2 V}{\partial S_{NG} \partial S_{QL}}$.

2.4 Results

2.4.1 Typical computational results

A computational experiment is carried out. The valuation result is shown in figure 2.5 and the control strategies $\theta_{NG}$ and $\theta_{QL}$ are shown in figure 2.6 and 2.7, respectively. From figure 2.5, we can see a dividing line like a ridge on which either coal or natural gas or any combination of them can be used and the energy cost will remain
the same. Alternatively, in the left region, coal is used and in the right natural gas is chosen as the fuel. When coal is used, $V$ will not change regardless of the magnitude of natural gas prices. This is why it is possible to observe a flat line at the top. You may notice that $V$ is not always nonnegative. When coal price is very small, $V$ is negative. This may look strange or unfamiliar at the first glance. The reason is when coal price is very low, the banker prefers to pay the utility operators to buy and store coal, benefitting from the convenience yield rather than deliver coal to the utility directly because when the coal price is higher, the banker has to pay more to satisfy the utility's fuel demand. Since natural gas has no convenience yield for this choice of parameter value, storing natural gas serves no purpose.

![Figure 2.5: A typical valuation surface](image)

Figure 2.5: A typical valuation surface
Figure 2.6: The control surface $\theta_{NG}$

Figure 2.7: The control surface $\theta_{QL}$
2.4.2 Validation results

2.4.2.1 Validation with simulated data

Monte Carlo method is used to simulate correlated natural gas and coal prices which follow a correlated Geometric Brownian motion (GMB) given in equation (2.12)-(2.14).

\[
\begin{align*}
\frac{dS_{NG}}{S_{NG}} &= \mu_{NG} dt + \sigma_{NG} dZ_1, \quad (2.12) \\
\frac{dS_{QL}}{S_{QL}} &= \mu_{QL} dt + \sigma_{QL} dZ_2, \quad (2.13) \\
dZ_1 dZ_2 &= \rho dt. \quad (2.14)
\end{align*}
\]

Correlated prices for 3 months (91 days) are modeled for 50 times. The results are shown in figure 2.8. A finite different method is used to solve equation (2.7) as talked in section 2.3.4 to obtain the corresponding \(\theta_{NG}\) and \(\theta_{QL}\) (through interpolation), which will be used to calculate the self-financing hedging portfolio \(V\) in equation (2.2) for validation. The values of the portfolio are shown in figure 2.9. From figure 2.9, we can see the values of the portfolios for each path finally goes to 0 at maturity. The mean value at time T is 51.05 $/MW (the theoretical value is 0). Comparing to the order of the magnitude \(5 \times 10^4\) $/MW, the error is about 0.1%, which is pretty small. Therefore we say the results validate our computation results. The error can come from numerical computation error and from interpolation error caused by Matlab function ‘interp2’ to obtain \(V, \theta_{NG}\) and \(\theta_{QL}\) based on the simulated prices.
Figure 2.8: Simulated stochastic price data

Figure 2.9: Validation results with simulated stochastic data
2.4.2.2 Validation with historical data

In this subsection, historical price data for natural gas and coal (2002-2011, see figure 2.10) are used to validate the simulation results. The data are separated into 5 periods, every two years as one period. Each period’s data are divided into to two parts. One part is the latest 91 days’ data, which is used for validation while the others are used to fit the parameters $\sigma_s$ and $\rho_s$ in the price models equation (2.8)-(2.10). The results are shown in table 2.3. The other parameters used in the validation process are the same as shown in table 2.1.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>0.1081</td>
<td>-0.0177</td>
<td>0.1899</td>
<td>0.2442</td>
<td>0.2165</td>
</tr>
<tr>
<td>$\sigma_{NG}$</td>
<td>0.0396</td>
<td>0.0308</td>
<td>0.0376</td>
<td>0.0335</td>
<td>0.0254</td>
</tr>
<tr>
<td>$\sigma_{QL}$</td>
<td>0.0134</td>
<td>0.0136</td>
<td>0.0112</td>
<td>0.0367</td>
<td>0.0157</td>
</tr>
</tbody>
</table>

The parameters in table 2.3 are substituted into equation (2.7), which is solved iteratively for each time step to obtain the corresponding $\theta$s through interpolation. The latest 91 days’ data of each period are used to perform validation with equation (2.2). The validation result is shown in figure 2.11. From figure 2.11, we can see 5 lines of $V$ values corresponding to the 5 periods (2002-2003, 2004-2005, 2006-2007, 2008-2009,
2010-2011) with different initial values all goes to 0 with time increasing. The average at
time T is -113.82 $/MW. Comparing with the theoretical value 0 $/MW, the relative
error is about 0.2% with the order of the magnitudes $6 \times 10^4 $/MW, which also
validates our simulation results. Similar to the reasons causing the error of the
validation results with simulated data, the error can come from numerical computation
error and interpolation error to obtain $V, \theta_{NG}$ and $\theta_{QL}$. Also it may be caused by
parameter fitting with real data.

Figure 2.10: Natural gas and coal prices (2002-2011)
2.5 Conclusion

Flexible utility operators are easily affected by volatile energy prices and expose themselves to the risks cause by price volatility. An energy swap is a useful tool, which is often adopted by utility operators to hedge the financial risks. Finding a 'fair' value for the operators to enter into the swap with a banker is important. In this chapter, a flexible fuel process that can operate with natural gas or coal with any combination is considered. A self-financing portfolio, which includes the flexible fuel process, cash invested in risk-free assets and the physical ownership of the energy commodities, is employed for the valuation of the energy swap. The 'fair' value and the optimal hedge is obtained by solving an HJB equation with PDE method. An interesting result is that the 'fair' value can be negative, which means instead of getting money from the utility operators to finance their cost, the banker would like to pay the operators to buy the
fuel directly in case of incurring a higher cost in the future. Correlated stochastic price data simulated with Monte Carlo method and historical data are both used for validation. The validation results support the simulation results.
CHAPTER 3:
VALUATION AND OPTIMIZATION OF ENERGY COMMODITY STORAGE WITH ONE STOCHASTIC VARIABLE

3.1 Introduction

Energy commodities, like natural gas, crude oil and coal, are indispensable to modern life and to industrial development. With rapid population increase and fierce competition in economic growth, it is becoming increasingly important to manage energy commodity storage. Energy commodity price is an important factor that affects storage operation strategies, which is mainly influenced by supply and demand. Tight supplies lead to great increase in price, which may take a toll on some industrial users. For example, a sudden increase in natural gas cost for small-business customers can cause severe results such as bankruptcy since they couldn’t afford the soared utility bills.

Efficient and reliable energy commodity storage can help mitigate the financial risks. First of all, it can help mitigate price fluctuation. For example, when demand is high and supply is tight, commodity price increases. Energy commodity can be released from storage to decrease the negative effects caused by the increased price. While when price is low, commodity is bought and stored, which is equivalent to an indirect
demand increase, preventing commodity price from further decrease. Furthermore, it can offer the energy storage facility owner/operators optimal storage operation strategies, lower investment risk, and increase profits. Managing commodity storage is an important and meaningful field of research and has attracted many researchers' attention [Thompon2009, Chen2007].

One of the earliest stories about successful commodity storage can be traced to Joseph’s interpretation of the Pharoah’s dream in the Bible. Pharoah dreamed “Seven heads of grain, healthy and good, were growing on a single stalk. After them, seven other heads of grain sprouted -- thin and scorched by the east wind.” Joseph successfully interpreted Pharoah’s dream and foreseen seven years’ harvest followed by seven years’ famine and then suggested Pharoah to collect huge quantities of grain and stored it under the good seven years for the future famine.

The theory of storage was originally developed and described by Working based on wheat study in 1933 [Working1933]. It was then extended by N. Kaldor who coined the term ‘convenience yield’ in 1939 [Kaldor1939]. In 1949, Working’s seminal article on economic exceptions, a measure of which can be future prices, opened the research gate of commodity storage. Specific research directions include how to measure convenience yield [Brennan1991, Milonas(1997a and 1997b), Schwartz1997, Heaney2002, Almansour2014]; try to find a proper energy commodity price model, for example, Deng proposed several mean-reversion jump diffusion models to describe energy commodity spot price [Deng2000]. Chen et al. proposed an one-factor regime-switching model for a risk-adjusted natural gas spot price and stated it was better than
an one-factor mean-reverting model [Chen2010]. He also talked about valuation method of commodity storage. Since our work will concentrate on this part, detailed review is talked about below.

Chen and Forsyth presented a semi-Lagrangian approach for solving natural gas storage valuation and optimal operation problem, which is a stochastic control problem. This approach can be used for a wide range of price models that exhibit mean-reverting seasonality dynamics and price jumps [Chen2007]. Boogert and Jong used Least Squares Monte Carlo for gas storage valuation [Boogert2008]. Thompson et al. used a PDE/PIDE method for the valuation and optimization of natural gas storage [Thompson2009]. Lai et al. used an approximate dynamic programming method coupled with Monte Carlo method to obtain the lower and upper bounds of storage [Lai2009]. Carmona et al. constructed an optimal switching approach for energy storage valuation with the natural gas dome storage and hydroelectric pumped storage as examples [Carmona2010]. Nadarajah developed a novel approximate dynamic programming method for the real option management of commodity storage by relaxing approximate linear programs obtained using value function approximations [Nadarajah2012]. Secomandi et al. investigated how the model error affected the valuation of the commodity assets and proposed an effective method to offset the negative effects of the model error [Secomandi2012].

However, in the research work of valuation and optimal operation of energy commodity storage, convenience yield and demand are often excluded from the storage model. Also, as for the valuation method, Monte Carol method is usually time-
consuming and computationally expensive. The semi-Lagrangian method is complicated for researchers outside this area to understand and to borrow the light. An explicit numerical method called ‘Total Diminishing Method’ employed by Thompson [Thompson2009] to solve the PDE/PIDE method is easy to understand, however, the artificial terms may result in bigger numerical error. The commodity storage problem is pointed out to be a stochastic control problem. However, what kind of stochastic control problem is it on earth? Those problems mentioned above will be investigated in this chapter.

The chapter is organized as follows. In section 3.2, a basic model for energy commodity storage including convenience yield and serving specific demand is proposed. The storage problem is then formulated into an HJB equation with a stochastic optimization problem embedded. Initial and boundary conditions are obtained mathematically and realistically. Easily understandable and efficient finite difference numerical schemes are given in detail. In section 3.3, an oil storage problem is considered as an example. Typical computational results, including valuation results and optimal control strategies, the effects of convenience yield and demand, and comparison results with other control methods, are shown. The control problem is further investigated. The chapter is concluded in section 3.4.

3.2 Problem formulation

In this section, a basic storage model including specific demand is proposed. A stochastic optimization problem based on the storage model is developed. A general
stochastic price model is used to simulate energy commodity price. PDE method is used to solve the formulated problem and numerical schemes are given in detail.

3.2.1 Storage model

A basic model for energy commodity storage serving constant demand $D$ is proposed as shown in figure 3.1. This model is different from those used in [Thompson2009] and [Chen2007], which don’t include demand. The storage inventory at time $t$, $I(t)$, is finite, $I(t) \in [0, I_{\text{max}}]$. Control variable $u(t)$ is also bounded, $|u(t)| \leq U$. We say when $u>0$, energy commodity is released while when $u<0$, it is stored, or else no commodity is stored or released. To reach a balance at the node, we need to purchase $D-u(t)$ commodity from the spot market at time $t$.

![Figure 3.1: Storage model with constant demand](image)

3.2.2 Mathematical derivation

Given the storage model in figure 3.1, we want to maximize the expected profit of the storage operation over the time period $[0, T]$. The objective function can be written as
\[
\max_u E\left[ \int_0^T e^{-r\tau} \left[ (u(\tau) - D(\tau)) S(\tau) + y I(\tau) \right] d\tau \right]
\]

subject to

\[
|u(\tau)| \leq U.
\]

In the objective function, \( u(t)S(t) \) represents the profit by selling commodity to the spot market; \( D(t)S(t) \) represents the cost of satisfying the demand; \( yI(t) \) is the profit caused by the convenience yield. Since the money today is not equivalent to that tomorrow, we have a discount factor \( e^{-rt} \) in the integral. The convenience yield \( y \) and the risk-free discount rate \( r \) are treated as constants. We suppose the spot price \( S \) follows a general stochastic model

\[
dS = a(S, t)dt + b(S, t)dZ,
\]

and the inventory \( I(t) \) satisfies an ordinary differential equation

\[
\frac{dI}{dt} = -u(t),
\]

where there is a minus sign in front of \( u(t) \). That is because when energy commodity is bought, \( u(t) \) is negative, while \( \frac{dI}{dt} \) is positive, therefore a minus sign is added in front of \( u(t) \).

Define a valuation function \( V(S, I, t) \),

\[
V(S, I, t) = \max_u E\left[ \int_t^T e^{-r(\tau-t)} \left[ (u(\tau) - D(\tau)) S(\tau) + y I(\tau) \right] d\tau \right]. \tag{3.1}
\]

Use the standard approach of constructing an HJB equation,
1. Separate the integral in the valuation function (3.1) into two parts. The first part is to do the integral from \( t \) to \( t+dt \), and the second part is from \( t+dt \) to \( T \) as shown in equation (3.2),

\[
V(S,I,t) = \max_u E \left[ \int_t^{t+dt} e^{-r(t-t')} [(u(\tau) - D)S(\tau) + yI(\tau)] d\tau \right. \\
\left. + \int_{t+dt}^{T} e^{-r(t-t')} [(u(\tau) - D)S(\tau) + yI(\tau)] d\tau \right].
\] (3.2)

2. Use rectangle method for the 1\(^{st}\) integral if we suppose \( dt \) is very small, and multiply \( e^{-rdt} \) outside the 2\(^{nd}\) integral and \( e^{rdt} \) inside,

\[
V(S,I,t) = \max_u E \left[ [(u(t) - D)S(t) + yI(t)] dt \right. \\
\left. + e^{-rdt} \int_{t+dt}^{T} e^{-r(t-t')} [(u(\tau) - D)S(\tau) + yI(\tau)] d\tau \right] \\
= \max_u E \left[ [(u(t) - D)S(t) + yI(t)] dt \right. \\
\left. + e^{-rdt} V(S + dS, I + dI, t + dt) \right].
\] (3.3)

3. Do Taylor expansion to the order of \( dt \) and using It\'s lemma,

\[
V(S,I,t) = \max_u E \left[ [(u - D)S + yI] dt \right. \\
\left. +(1-\frac{rdt}{2})[V + a \frac{\partial V}{\partial S} dt + b \frac{\partial V}{\partial S} dZ] \\
\left. + \frac{1}{2} b^2 \frac{\partial^2 V}{\partial S^2} dt - u \frac{\partial V}{\partial I} dt + \frac{\partial V}{\partial t} dt) \right].
\] (3.4)

4. Take expectations and divide both sides of equation (3.4) by \( dt \) and obtain

\[
-\frac{\partial V}{\partial t} = \frac{1}{2} b^2 \frac{\partial^2 V}{\partial S^2} + a \frac{\partial V}{\partial S} - rV - DS + yI + \max_u [u(S - \frac{\partial V}{\partial I})].
\] (3.5)

5. Change \( t \) to \( t' = T-t \), equation (3.5) becomes

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There is an optimization problem embedded in the PDE (3.6)

\[
\max_u [u(S - \frac{\partial V}{\partial I})]
\]

subject to

\[|u(t)| \leq U.\]

The optimal \(u\) satisfies

\[
\frac{\partial V}{\partial t'} = \frac{1}{2} b^2 \frac{\partial^2 V}{\partial S^2} + a \frac{\partial V}{\partial S} - rV - DS + yI + \max_u [u(S - \frac{\partial V}{\partial I})].
\]  

(3.6)

3.2.3 Initial and boundary conditions

To solve the PDE (3.6), initial and boundary conditions are needed.

In \(t'\), the initial condition is easily to be obtained from equation (3.1). In equation (3.1), if \(t=T\),

\[
V(S, I, T) = 0.
\]

Since \(t'=T-t\), when \(t=T\), \(t'=0\), then we have

\[
V(S, I, 0) = 0.
\]

Since \(V\) depends on the second derivative with respect to (w.r.t.) \(S\), two boundary conditions are needed in \(S\). Thompson et al. pointed out that the boundary conditions depend on the specific spot price model. In most cases, the boundary conditions below are sufficient [Thompson2009].

\[
V_{SS} = 0 \quad \text{for } S \text{ large},
\]
\[ V_{SS} = 0 \quad \text{for} \ S \to 0. \]

The reason is when S is very small/large, the control strategy is always to buy/sell. A small change in S will not change the optimal control strategy. So V must change linearly w.r.t. S near the boundaries.

Equation (3.6) is hyperbolic in I since there is only the first derivative w.r.t. I. One equality boundary for I is enough normally. However, in our work, instead of using one equality boundary condition, two inequalities are used in I domain,

\[ u \geq 0 \quad \text{for} \ I = I_{\max}, \]
\[ u \leq 0 \quad \text{for} \ I = 0, \]

which mean when the reservoir is full \( (I=I_{\max}) \), no more energy commodity can be stored, therefore \( u \geq 0 \), while when the reservoir is empty \( (I=0) \), no commodity can be released, so we have \( u \leq 0 \).

3.2.4 Numerical schemes

We use a standard 1\(^{\text{st}}\)-order accurate explicit finite difference method for the discretization in \( t' \), S, I, which is much simpler than the total variation diminishing (TVD) scheme used by Thompson et al. [Thompson2009].

Let \( V_{i,j}^k \) represent the value of V at the \( i^{\text{th}} \) grid point value of S, the \( j^{\text{th}} \) grid point value of I at time step k. \( i=1, 2, ..., M-1, M; j=1, 2 ..., J-1, J. \) M and J are the total numbers of grid points in S and in I, respectively.

In \( t' \), the discretization formula is
The first partial derivatives w.r.t. $S$ for the interior points are calculated using the upwind method, which depends on the sign of the coefficient $a$,

\[
\frac{\partial V^k_{i,j}}{\partial t} = V^{k+1}_{i,j} - V^k_{i,j},
\]

\[
\frac{\partial V^k_{i,j}}{\partial S} = \frac{V^k_{i,j} - V^k_{i+1,j}}{\delta S} \quad \text{if } a \leq 0,
\]

\[
\frac{\partial V^k_{i,j}}{\partial S} = \frac{V^k_{i+1,j} - V^k_{i,j}}{\delta S} \quad \text{if } a > 0,
\]

where $\delta S$ is the step size in $S$ direction.

The discretization formula for the interior points of the second partial derivative w.r.t. $S$ is central discretization scheme

\[
\frac{\partial^2 V^k_{i,j}}{\partial S^2} = \frac{V^k_{i+1,j} - 2V^k_{i,j} + V^k_{i-1,j}}{(\delta S)^2}.
\]

For the points on the boundaries $(i=1, M)$, the formulae are obtained by making good use of the boundary conditions

\[
\frac{\partial^2 V^k_{1,j}}{\partial S^2} = 0,
\]

\[
\frac{\partial^2 V^k_{M,j}}{\partial S^2} = 0.
\]

For the first derivative w.r.t. $I$, the formulae are similar to those in $S$, which depends on the sign of the coefficient $u$ in front of $\frac{\partial V}{\partial I}$,

\[
\frac{\partial V^k_{i,j}}{\partial I} = \frac{V^k_{i,j+1} - V^k_{i,j}}{\delta I} \quad \text{if } u \leq 0,
\]

\[
\frac{\partial V^k_{i,j}}{\partial I} = \frac{V^k_{i,j+1} - V^k_{i,j}}{\delta I} \quad \text{if } u > 0.
\]
\[ \frac{\partial V^k_{i,j}}{\partial I} = \frac{V^k_{i,j} - V^k_{i,j-1}}{\delta I} \text{ if } u > 0. \]

3.3 Typical computational results

An oil storage problem is considered as an example. Suppose there is an oil storage facility. The maximum releasing/injecting rate \( U = 50 \) bbl/day. The risk-free discount rate \( r \) is 10% per year. The demand \( D \) is 0 bbl/day. The convenience yield \( y \) is 0.2 $/bbl/day. The maximum storage capacity \( I_{\text{max}} \) is 1500 bbl. The spot price \( S \) follows a mean-reverting process

\[ dS = 0.25(100 - S) \, dt + 0.2 S dZ. \] (3.7)

Substitute the price model (3.7) and the given parameters into equation (3.6) and get

\[ \frac{\partial V}{\partial t'} = \frac{1}{2} 0.04S^2 \frac{\partial^2 V}{\partial S^2} + 0.25(100 - S) \frac{\partial V}{\partial S} - 0.1/365V - 50S + 0.2I + \max_u [u(S - \frac{\partial V}{\partial I})], \]

where \( u \) is obtained by solving the optimization problem

\[ \max_u [u(S - \frac{\partial V}{\partial I})] \] (3.8)

subject to

\[ |u(t)| \leq U. \]

Carefully look at the objective function (3.8) and it is not difficult to find out
Actually we can’t decide the optimal control by simply evaluating the sign of \( S - \frac{\partial V}{\partial I} \) at each grid point \( V_{i,j}^k \) since we don't know what kind of differencing numerical schemes we should use to discretize \( \frac{\partial V}{\partial I} \), which depends on the control \( u(t) \). How can this problem be solved then? To solve the problem, instead of obtaining \( u(t) \) and the sign of \( S - \frac{\partial V}{\partial I} \) separately, we can get the optimal value of the objective function by comparing three terms \( U(S - \frac{\partial V}{\partial I}), -U(S - \frac{\partial V}{\partial I}), \) and \( 0 \). The optimal value is the maximum of them and meanwhile the optimal control is decided.

3.3.1 Valuation results

After running the simulation until the final time \( T=63 \) days, we get the results shown in figure 3.2 and 3.3. Figure 3.2 shows the valuation surface and figure 3.3 is the corresponding control strategy surface.
Figure 3.2: The typical valuation surface for 2D storage problem
(U=50 bbl/day, I_{max}=2000 bbl, r=0.1 per year, D=0 bbl/day, y=0.2 $/bbl/day, dS=0.25(100-S)dt+0.2SdZ, T=63 day.)

Figure 3.3: The typical control surface for 2D storage problem
(U=50 bbl/day, I_{max}=2000 bbl, r=0.1 per year, D=0 bbl/day, y=0.2 $/bbl/day, dS=0.25(100-S)dt+0.2SdZ, T=63 day.)
As for the valuation surface (figure 3.2), we can see when oil price is extremely high and the reservoir is full, the value of the storage facility reaches the maximum. When the reservoir is empty and oil price is at its highest value, the value of the facility reaches its minimum. That's because when oil price is extremely high and the storage tank is full, we try to sell the commodity as fast as we can to make the biggest profit; when oil price is extremely high and the storage tank is empty, nothing can be sold and it is not worth buying the oil to store since the price is very high. Therefore the value of the facility reaches its minimum. For the rest of time, when oil price is relatively low, the utility owner/operators buy oil from the spot market and store it for 'sell high', while when oil price is relatively high, they keep the storage tank where it is since it is not worth for storage.

As for the optimal control surface (figure 3.3), the result is basically consistent with our intuition, 'buy low, sell high'. When oil price is low, we buy the oil from the spot market and store it; when oil price is high, we sell the stored oil to the spot market to make profits. At the boundaries (either the storage tank is empty or full), the control strategies are a little different. When the storage tank is empty and price is relatively high, the controls are 0. The same happens when the reservoir is full and price is relatively low. That’s because when the storage tank is empty, nothing can be released even if the oil price is extremely high. Similarly, when the storage tank is full, no more oil can be stored even if the price is extremely low. Generally the control is 'bang-bang' control, which means oil is either to be bought or sold. The switching line of the control
is not at the mean-reverting level $S=100 \$/bbl, which may be dragged down by an Itô correction.

We further investigated the control policy. Figure 3.4 shows how the control policy switches with price, inventory and time change. The red line carries all the information. It is like a needle with thread going through the green zero control surface. When price is relatively low, $u=-U$, which is ‘to buy’; when price is relatively high, $u=U$, which is ‘to sell’. The control policy ends up being a stochastic sliding mode control. The chattering effect may be reduced or eliminated by reducing the amplitude of the signum function or replacing the signum function by a high-slope saturation function [Khalil2002].

Figure 3.4: The sliding mode control policy
(U=50 bbl/day, I_{max}=2000 bbl, r=0.1 per year, D=0 bbl/day, y=0.2 \$/bbl/day, dS=0.25(100-S)dt+0.2SdZ, T=63 day.)
3.3.2 Parameter effects

In this section, the effects of convenience yield and demand are investigated.

Figure 3.5 shows the effect of the convenience yield $y$ on the valuation function $V$. With $y$ increasing, the value of the storage facility becomes larger and larger. The larger the inventory is, the bigger the value of the facility is. The result shows the convenience yield in establishing incremental value for energy storage facility.

Figure 3.5: The effect of convenience yield on the value surfaces

Further take a look at the effect of $y$ on the value of the facility, it follows a linear relationship as shown in figure 3.6.
Figure 3.6: The relationship of value $V$ and convenience yield $y$

Fit the linear relationship of $V$ vs. $y$ as shown in figure 3.7 and get $V=(0.6206*y+2.4770)\times10^5$, where $R^2=0.9994$ and the root mean squared error is 443.5372 $\$. 

Figure 3.7: The fitted line of $V$ vs. $y$
Figure 3.8 shows the effect of demand $D$ on the valuation function $V$. From figure 3.8, we can see the value of the oil facility decreases with demand increasing. That’s because the higher demand, the more cost we need to spend on satisfying the demand and the less profit we can make. Further take a look at the demand effect on the value of the facility, it follows a linear relationship as well as shown in figure 3.9.

Figure 3.8: The effect of demand on the value surfaces
Figure 3.9: The relationship of value V and demand D

Further check the relationship of value V and demand D to see if it is strictly linear, we get $V=(-0.0675*D+2.5986) \times 10^5$, where $R^2 = 1.0000$, the root mean squared error is $6.8885 \times 10^{-10}$. The result shows V is strictly linearly related with D.

Figure 3.10: The fitted line of V vs. D
3.3.3 Comparison

3.3.3.1 Comparison with no control policy

The valuation result with optimal controls strategies is compared to the case without no controls. The comparison result is shown in figure 3.11. From this figure we can see the valuation surface with the optimal control is much higher than that without any control. Therefore we say it is really profitable to operate the facility following the optimal control strategies.

![Comparison of valuation surfaces with and without optimal controls](image)

**Figure 3.11**: Comparison of valuation surfaces with and without optimal controls

(U=100 bbl/day, I_{max}=1500 bbl, r=0.1 per year, D=0 bbl/day, y=0.2 $/bbl/day, dS=0.25(100-S)dt+0.2SdZ, T=63 day.)
3.3.3.2 Comparison with threshold method

The valuation surface with optimal controls is further compared to the surface with threshold control method. Choose the mean-reverting value 100$/bbl as the threshold value. If the oil price is higher than 100 $/bbl, it is sold; if it is lower than 100 $/bbl, it is bought, or else neither bought nor sold. The comparison result is shown in figure 3.12. From this figure we can see the valuation surface with the optimal controls is much higher than the surface with threshold method. Therefore it is really beneficial to operate following the optimal control strategies.

Figure 3.12: Comparison of valuation surfaces with optimal controls and with threshold controls
(U=100 bbl/day, S_{threshold}=100 $/bbl/day, I_{max}=1500 bbl, r=0.1 per year, D=0 bbl/day, y=0.2 $/bbl/day, dS=0.25(100-S)dt+0.2SdZ, T=63 day.)
3.4 Conclusion

In this chapter, a basic model for energy commodity storage servicing known demand is proposed. The storage holds a finite inventory of an energy commodity, such as coal, natural gas, or oil. It is subjected to boundaries on the rate at which the energy commodity can be stored or released. An example of oil storage problem is given. The optimal operation and valuation of energy storage facility is obtained as the solution to an HJB equation. The optimal control 'buy low, sell high' is the same as expected. 'Bang-bang' optimal control is obtained based on a mean-reverting spot price model. The switching line of the control strategy is at a lower position than the mean-reverting lever, which is probably dragged down by an Ito correction. Further check the control problem, it is found as a stochastic sliding mode control problem, which is novel and worth further investigation. As for this storage model, not only mean-reverting price model can be used in similar problems, but various energy commodity price models, like models with jumps, can be adapted to solve the corresponding HJBs. The results also show the convenience yield in establishing incremental value for energy storage facilities and the effect of the demand on the valuation function, which is with the demand increasing, the value of the facilities decreases and vice versa. The linear relationships between the facility value and convenience yield/demand are fitted. Besides, optimal operation of the storage facility provides the owners/operators of energy utility with a means for gaining more profits by comparing with other control strategies, like no control and threshold control method.
CHAPTER 4:
VALUATION AND OPTIMIZATION OF ENERGY COMMODITY STORAGE WITH TWO
STOCHASTIC VARIABLES

4.1 Introduction

In reality, demand is usually unpredictable [Wallace2003]. Murillo-Sánchez et al. pointed out “models for centrally-dispatched storage and time-flexible demands allow for optimal tradeoffs between arbitraging across time, mitigating uncertainty and covering contingencies” and presented a stochastic optimization framework for operations and planning of an electricity network with RHC method [Murillo2013]. Powell et al. also addressed the problem of capturing energy demand uncertainty in modeling energy storage and proposed a modeling and algorithmic strategy based on the framework of approximate dynamic programming [Powell2012]. Considering from utility customers’ perspectives, Oh et al. proposed a new decision process to assess the impact pertaining to the energy demand response program participation with scenario tree methods and suggested a stochastic energy demand model where the energy demands were the stochastic variables following an n-discretized distribution [Oh2011]. Zhou et al. investigated the impacts of demand and supply uncertainty on the optimal design of distributed energy systems and proposed a two-stage stochastic programming
model for the optimal design of distributed energy systems with genetic algorithm and Monte Carlo method [Zhou2013].

Besides stochastic demand, energy commodity price is often correlated with other variables, like convenience yield, volatility and another energy commodity price as stated in section 1.3.3.2. In this chapter, we further expand our storage model by including one more stochastic variable, like stochastic demand, volatility, convenience yield or another energy commodity price, to make our model more versatile.

The chapter is organized as follows. In section 4.2, storage models and mathematical formulation processes for the cases with two stochastic variables are introduced. Numerical schemes are exhibited in section 4.3. The problem including stochastic demand as the second variable is solved as an example in section 4.4. Numerical convergence of computation results is checked with the numerical convergence order calculation by choosing different computation grid points in section 4.5 and the chapter is concluded in section 4.6.

4.2 Mathematical modeling

4.2.1 Stochastic demand

4.2.1.1 Storage model

The storage model with stochastic demand is similar to the 2D’s with constant demand (figure 3.1) except for having stochastic demand D(t) instead of known demand D. The model is shown in figure 4.1.
4.2.1.2 Problem formulation

The mathematical derivation process is also similar to 2D’s (See section 3.3).

Given the storage model in figure 4.1, we want to maximize the expected profit of the storage operation over the period \([0, T]\). The objective function can be written as

\[
\max_u E\left[ \int_0^T e^{-r\tau} \left( (u(\tau) - D(\tau)S(\tau) + yI(\tau)) \right) d\tau \right]
\]

subject to

\[|u(\tau)| \leq U.\]

Suppose the spot price \(S(t)\) and the demand \(D(t)\) follow a general stochastic model, respectively. The two stochastic models are correlated with correlation coefficient \(p\). Then

\[
dS = a(S, t)dt + b(S, t)dZ_1,
\]

\[
dD = p(D, t)dt + q(D, t)dZ_2,
\]

\[
dZ_1dZ_2 = \rho dt.
\]
The inventory $I(t)$ satisfies an ordinary differential equation

$$\frac{dI}{dt} = -u(t).$$

Define a valuation function $V(S,I,D,t)$,

$$V(S,I,D,t) = \max_u E\left[ \int_t^T e^{-r(t-\tau)}[(u(\tau) - D(\tau))S(\tau) + yI(\tau)]d\tau \right]. \quad (4.1)$$

Using the standard approach of constructing an HJB equation as shown in section 3.3.2 and obtain a PDE

$$\frac{\partial V}{\partial t'} = \frac{1}{2} b^2 \frac{\partial^2 V}{\partial S^2} + a \frac{\partial V}{\partial S} + \frac{1}{2} q^2 \frac{\partial^2 V}{\partial D^2} + p \frac{\partial V}{\partial D} + \rho b q \frac{\partial^2 V}{\partial S \partial D} - rV - DS + yI + \max_u [u(S - \frac{\partial V}{\partial I})], \quad (4.2)$$

where $t' = T - t$.

4.2.1.3 Initial and boundary conditions

To solve the PDE (4.2), initial and boundary conditions are needed.

In $t'$, the initial condition is the same as we used for the 2D’s problem with the same reason (see section 3.2.3)

$$V(S,D,I,0) = 0.$$

Since $V$ depends on the second derivative w.r.t. $S$ and $D$, two boundary conditions are needed for each variable. For both of the two variables, we use the 2nd derivatives equal to 0 when each variable is very large/small,

$$V_{SS} = 0 \quad \text{for } S \text{ large},$$

$$V_{SS} = 0 \quad \text{for } S \to 0.$$

$$V_{DD} = 0 \quad \text{for } D \text{ large},$$

$$V_{DD} = 0 \quad \text{for } D \to 0.$$

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\[ V_{D_0} = 0 \quad \text{for} \ D \to 0. \]

In S, the reason is the same as we explained in section 3.2.3. In D, the reason is similar.

No matter demand is very large or small, we always want to satisfy the demand and the control strategy will not change. Therefore we use the 2\textsuperscript{nd} derivatives are 0 near the boundaries.

In I, we also use the two inequalities as we used for 2D’s with the same reasons as explained in section 3.2.3,

\[ u \geq 0 \quad \text{for} \ I = I_{\text{max}}, \]

\[ u \leq 0 \quad \text{for} \ I = 0. \]

4.2.2 Stochastic convenience yield

4.2.2.1 Storage model

The storage model is the same as shown in figure 4.1.

4.2.2.2 Problem formulation

The derivation process for stochastic convenience yield \( y(t) \) is the same as stochastic demand. The HJB equation is shown in equation (4.3).

\[
\frac{\partial V}{\partial t} = \frac{1}{2} b_s \frac{\partial^2 V}{\partial S^2} + a \frac{\partial V}{\partial S} + \frac{1}{2} q_s \frac{\partial^2 V}{\partial y^2} + p \frac{\partial V}{\partial y} + \rho b_q \frac{\partial^2 V}{\partial S \partial y} - rV - DSyI + \max[u(S - \frac{\partial V}{\partial I})] \quad \text{(4.3)}
\]
4.2.2.3 Initial and boundary conditions

The boundary conditions used for stochastic convenience yield are the same with stochastic demand.

In $t'$, the initial condition is

$$V(S,D,I,0) = 0.$$ 

The two boundary conditions for $S$ and $y$ are

$$V_{SS} = 0 \quad \text{for } S \text{ large},$$

$$V_{SS} = 0 \quad \text{for } S \to 0,$$

$$V_{yy} = 0 \quad \text{for } y \text{ large},$$

$$V_{yy} = 0 \quad \text{for } y \to 0.$$ 

In $S$, the reason is the same as we explained in section 3.2.3. In $y$, the reason is similar. No matter convenience yield is very large/small, we always want to buy/sell the commodity to make the most profits by making good use of the convenience yield and the control strategy stands still. So we use the $2^{nd}$ derivatives equal zero when $y(t)$ is very small/large.

In $I$, we also use the two inequalities as we used for $2D$'s,

$$u \geq 0 \quad \text{for } I = I_{\text{max}},$$

$$u \leq 0 \quad \text{for } I = 0.$$
4.2.3 Stochastic volatility

4.2.3.1 Storage model

The storage model is the same as shown in figure 4.1.

4.2.3.2 Problem formulation

The derivation process of the problem for stochastic volatility $v(t)$ is also the same as the stochastic demand. The result is shown in equation (4.4).

$$\frac{\partial V}{\partial t'} = \frac{1}{2} b^2 \frac{\partial^2 V}{\partial S^2} + a \frac{\partial V}{\partial S} + \frac{1}{2} q^2 \frac{\partial^2 V}{\partial v^2} + p \frac{\partial V}{\partial v} + \rho bq \frac{\partial^2 V}{\partial S \partial v} - rV - DS + yI + max[u(S - \frac{\partial V}{\partial I})] \quad (4.4)$$

4.2.3.3 Initial and boundary conditions

The boundary conditions used for stochastic convenience yield are a little different from those used for stochastic demand.

In $t'$, the initial condition is

$$V(S, D, I, 0) = 0.$$ 

The two boundary conditions for $S$ and $v$ are

$$V_{SS} = 0 \quad \text{for } S \text{ large},$$

$$V_{SS} = 0 \quad \text{for } S \to 0.$$ 

$$V_v = 0 \quad \text{for } v \text{ large},$$

$$V_v = 0 \quad \text{for } v \to 0.$$
In S, the reason is the same as we explained in section 3.2.3. In v, since V is not directly related with v, therefore the $1^{st}$ derivatives w.r.t v equal to zero.

In I, we also use the two inequalities as we used for 2D’s,

$$u \geq 0 \quad \text{for } I = I_{\text{max}},$$

$$u \leq 0 \quad \text{for } I = 0.$$

4.2.4 Another energy commodity

4.2.4.1 Storage model

The storage model with two energy commodities as shown in figure 4.2 is different from that for stochastic demand/volatility/convenience yield. In figure 4.2, energy facility owner/operators import coal and natural gas with flow rates $f_{\text{QL}}$ and $f_{\text{NG}}$ from the spot market to satisfy a certain demand $D$. $u(t)$ is the control variable bounded by the maximum releasing/injecting rate $U$. In this model, coal can be stored while natural gas can’t, which is what happens in reality.

![Figure 4.2: Storage model with two commodities](image)

$u(t) \leq U$
4.2.4.2 Problem formulation

The mathematical derivation process is also similar to 2D’s (see section 3.2.2). Given the storage model in figure 4.2, we want to maximize the expected profit of the storage operation over the period \([0, T]\). The objective function can be written as

\[
\max_u E \left[ \int_0^T e^{-rt} \left( -f_{NG}S_{NG}(\tau) - f_{QL}S_{QL}(\tau) + yI(\tau) \right) d\tau \right]
\]  

(4.5)

subject to

\[ |u(t)| \leq U. \]

In the objective function (4.5), \(f_{NG}S_{NG}(t)\) and \(f_{QL}S_{QL}(t)\) represent the costs of buying natural gas and coal from the spot market to satisfy the constant demand at time \(t\); \(yI(t)\) is the profit brought by convenience yield; \(e^{-rt}\) is the discount coefficient.

Suppose the spot prices \(S_{NG}(t)\) and \(S_{QL}(t)\) follow a general stochastic model in equation (4.6)-(4.8). The two stochastic models are correlated with correlation coefficient \(\rho\),

\[
dS_{NG} = a(S_{NG}, t)dt + b(S_{NG}, t)dZ_1,
\]

(4.6)

\[
dS_{QL} = a(S_{QL}, t)dt + b(S_{QL}, t)dZ_2,
\]

(4.7)

\[
dZ_1dZ_2 = \rho dt.
\]

(4.8)

The inventory \(I(t)\) still satisfies the ordinary differential equation

\[
\frac{dI}{dt} = -u(t).
\]

We also have the mass balance

\[
\frac{f_{NG}}{H_{NG}} + \frac{f_{QL}}{H_{QL}} + u = D,
\]
where $H_{NG}$ and $H_{QL}$ are the heating rates of natural gas and coal, respectively.

Define a valuation function $V(S_{QL}, S_{NG}, I, t)$,

$$V(S_{QL}, S_{NG}, I, t) = \max_u E \left[ \int_t^T e^{-(r(t)\tau)} \left( -\left( D - \frac{f_{QL} + u(\tau)}{H_{QL}} \right) H_{NG} S_{NG}(\tau) - f_{QL} S_{QL}(\tau) + yI(\tau) \right) \, d\tau \right].$$  \hspace{1cm} (4.9)

Using the standard approach of constructing an HJB equation as shown in section 3.2.2 and obtain the HJB in equation (4.10).

$$\frac{\partial V}{\partial t} = \frac{1}{2} b^2 \frac{\partial^2 V}{\partial S_{NG}^2} + a \frac{\partial V}{\partial S_{NG}} + \frac{1}{2} q^2 \frac{\partial^2 V}{\partial S_{QL}^2} + p \frac{\partial V}{\partial S_{QL}} + \rho b q \frac{\partial^2 V}{\partial S_{NG} \partial S_{QL}} - rV - DH_{NG} S_{NG} + yI$$ 

$$+ \max_{u, f_{QL}} \left( u \left( \frac{H_{NG} S_{NG}}{H_{QL}} - \frac{\partial V}{\partial I} \right) + f_{QL} \left( \frac{H_{NG} S_{NG}}{H_{QL}} - S_{QL} \right) \right)$$  \hspace{1cm} (4.10)

The embedded optimization problem is

$$\max_{u, f_{QL}} \left( u \left( \frac{H_{NG} S_{NG}}{H_{QL}} - \frac{\partial V}{\partial I} \right) + f_{QL} \left( \frac{H_{NG} S_{NG}}{H_{QL}} - S_{QL} \right) \right)$$

subject to

$$|u(t)| \leq U,$$

$$f_{QL} + u \geq 0,$$

$$D - \frac{f_{QL} + u}{H_{QL}} \geq 0.$$

which is a linear programming problem with its possible solution plotted in figure 4.3.

The optimal solution locates at one of the vertices, A, B, C, or D, where at A, $u=-U$, $f_{QL}=D H_{QL} + U$; at B, $u=-U$, $f_{QL}=U$; at C, $u=U$, $f_{QL}=D H_{QL} - U$; and at D, $u=U$, $f_{QL}=U$.
4.2.4.3 Initial and boundary conditions

To solve the PDE (4.10), initial and boundary conditions are needed.

In t, the terminal condition is $V(S_{QL},S_{NG},I,T)=0$, which is obtained from the valuation function (4.9). Since $t'=T-t$, we have $V(S_{QL},S_{NG},I,0)=0$ for $t'$.

For both $S_{NG}$ and $S_{QL}$, we use $V_{SS}=0$ with the same reasons as stated in the 2D problem (see section 3.2.3).

In I, we also use $u \geq 0$ when $I=I_{max}$; and $u \leq 0$ when $I=0$, which is the same as used in the 2D’s problem.

4.3 Numerical schemes

The 1st-order accurate explicit finite difference method is employed for the discretization in $t'$, S, D, and I. For the 1st derivative, upwind numerical scheme is used;
for the 2nd derivatives, central approximation is employed; and for t’, forward method is adopted.

Take the stochastic demand 3D problem as an example.

Let \( V_{i,j,m}^k \) represent the value of \( V \) at the \( i \)th grid point value of \( S \), the \( j \)th grid point value of \( I \), \( m \)th grid point value of \( D \) at time step \( k \), where \( i=1, 2, \ldots, II-1, II; j=1, 2 \ldots, J-1, J; m=1, 2, \ldots, M-1, M \). \( II, J \) and \( M \) are the total numbers of grid points in \( S \), \( I \) and \( D \), respectively.

In \( t’ \), the discretization formula is

\[
\frac{\partial V_{i,j,m}^k}{\partial t'} = \frac{V_{i,j,m}^{k+1} - V_{i,j,m}^k}{\delta t'}. 
\]

The first partial derivatives w.r.t. \( S \) (\( S_{QL} \) and \( S_{NG} \)) for the interior points will be calculated using

\[
\frac{\partial V_{i,j,m}^k}{\partial S} = \frac{V_{i,j,m}^k - V_{i-1,j,m}^k}{\delta S}, \text{ if } a \leq 0,
\]

\[
\frac{\partial V_{i,j,m}^k}{\partial S} = \frac{V_{i+1,j,m}^k - V_{i,j,m}^k}{\delta S}, \text{ if } a > 0.
\]

where \( \delta S \) is the step size in \( S \) direction.

The discretization formula for the interior points of the second partial derivative w.r.t. \( S \) is

\[
\frac{\partial^2 V_{i,j,m}^k}{\partial S^2} = \frac{V_{i+1,j,m}^k - 2V_{i,j,m}^k + V_{i-1,j,m}^k}{(\delta S)^2}.
\]
For the points on the boundaries (i=1,II), the formulae are obtained by making good use of the boundary conditions

\[ \frac{\partial^2 V^k_{i,j,m}}{\partial S^2} = 0, \]
\[ \frac{\partial^2 V^k_{II,j,m}}{\partial S^2} = 0. \]

Similar discretization formulae are used for D. For the 1st derivative w.r.t. D, the formulae are

\[ \frac{\partial V^k_{i,j,m}}{\partial D} = \frac{V^k_{i,j,m} - V^k_{i,j,m-1}}{\delta D}, \text{ if } p \leq 0, \]
\[ \frac{\partial V^k_{i,j,m}}{\partial D} = \frac{V^k_{i,j,m+1} - V^k_{i,j,m}}{\delta D}, \text{ if } p > 0, \]

where \( \delta D \) is the step size in D direction.

For the 2nd derivatives w.r.t D, the formulae are

\[ \frac{\partial^2 V^k_{i,j,m}}{\partial D^2} = \frac{V^k_{i,j,m+1} - 2V^k_{i,j,m} + V^k_{i,j,m-1}}{(\delta D)^2}, \text{ for } m=2,\ldots,M-1, \]
\[ \frac{\partial^2 V^k_{i,j,1}}{\partial S^2} = 0, \]
\[ \frac{\partial^2 V^k_{i,j,M}}{\partial S^2} = 0. \]

For the first derivative w.r.t. I, the formulae are similar to those in S, which depends on the sign of the coefficient \( u \) in front of \( \frac{\partial V}{\partial I} \),

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\[
\frac{\partial V^k_{i,j,m}}{\partial I} = \frac{V^k_{i,j+1,m} - V^k_{i,j,m}}{\delta I}, \text{ if } u \leq 0,
\]
\[
\frac{\partial V^k_{i,j,m}}{\partial I} = \frac{V^k_{i,j,m} - V^k_{i,j-1,m}}{\delta I}, \text{ if } u > 0.
\]

4.4 Typical computation results

The same parameters and price model are used for 3D problem as for 2D except that the stochastic demand follows a stochastic model \( dD = 0.3(50-D)dt + 0.25DdZ \). The valuation result and the control strategy surfaces when \( D(t)=0 \) are shown in figure 4.4 and 4.5, respectively. The maximum norm of the absolute value of the difference of the two value surfaces is \( 2.91 \times 10^{-11} \), which shows the 3D results are consistent with 2D’s.

Figure 4.4: The valuation surface with stochastic demand \( D=0 \) (\( U=100 \text{ bbl/day}, l_{max}=1500 \text{ bbl}, r=0.1 \text{ per year}, y=0.2 \$/bbl/day, \]
\( dS=0.25(100-S)dt+0.2SdZ, T=63 \text{ day.} \))
4.5 Numerical Convergence

The numerical convergence order of the 3D computation is further checked. By choosing the grid number to be 20, 40, 80, respectively, the numerical convergence order is

\[
\frac{\log e_1 - \log e_2}{\log h_1 - \log h_2} = 0.69 \approx 1 ,
\]

where

\[
e_1 = \frac{\sum_{i=1}^{N} |V_{1i} - V_{2i}|}{N},
\]

\[
e_2 = \frac{\sum_{i=1}^{N} |V_{2i} - V_{3i}|}{N}.
\]
V1, V2 and V3 are the correspondent V values when grid number is 20, 40, and 80, respectively. h₁ and h₂ are the corresponding grid sizes (i.e., if e₁ is calculated with the grid size for V1(V2), then e₂ will be calculated using the grid size in V2(V3)). The accuracy of the numerical method (1ˢᵗ order) is consistent with the numerical convergence order (approximate to 1). Therefore, we say the 3D computation converges.

4.6 Conclusion

In this chapter, the storage problem is further extended into 3D problems by including one more stochastic variable, like stochastic demand, convenience yield, volatility or another commodity price. Storage models are the same for all these stochastic variables except for that including another energy commodity price. HJB equations are obtained following the same procedures in 2D’s. Initial and boundary conditions are given and explained for each 3D problem and a finite difference method is employed to discretize each derivative in the HJBs. The 3D problem with stochastic demand is solved as an example with a first-order accurate numerical method and the results are validated by comparing with 2D’s and by checking the numerical convergence order. The 3D results are consistent with 2D’s and the numerical convergence order 0.69 is approximate to the numerical computation order 1. Therefore we say the 3D computation is convergent.
CHAPTER 5:
CONCLUSION AND FUTURE DIRECTIONS

5.1 Conclusion

Energy commodity price fluctuation resulted from unbalanced supply and demand and other unpredicted external factors causes financial risks. To mitigate or avoid the potential risks, efficient energy risk management is necessary for energy utility owner/operators. In this dissertation, two problems in energy risk management are investigated. One is ‘Valuation of an energy swap for a flexible fuel process’ and the other is “Valuation and optimization of energy commodity storage”.

As for the energy swap problem, the purpose is to help energy utility owner/operators decide a ‘fair’ value to accept the contract with a banker. The energy swap was evaluated by constructing a self-financial portfolio including a flexible fuel process, cash invested in risk-free assets and the ownership of natural gas and coal inventories. By using PDE method, the ‘fair’ value was obtained by solving an HJB equation with a finite difference method. Hedging strategies were decided as the optimal control strategies. Both simulated stochastic and historical price data were used for validation. The validation results were in agreement with the computation results.
As for the storage problem, a storage model with constant demand was first proposed. An HJB equation with embedded stochastic optimization problem was developed. With PDE method, the optimal storage operations were obtained by solving the HJB. An oil storage problem was given as an example. The oil price was model with a mean-reverting price model. The obtained optimal control strategy was ‘Bang-Bang’ control -- ‘buy low, sell high’. The switching of the control strategy didn’t locate on the mean-reverting level, which might be cause by Itô correction. The control problem was further investigated to be a stochastic sliding mode control problem, which is novel and worth further investigation. The effects of important parameters, demand and convenience yield, were also evaluated. It was found that demand and convenience yield were negatively and positively proportional to the value of the facility, respectively. The valuation results with optimal controls were also compared to the cases with no controls and with threshold controls. The results showed the values with optimal controls were much higher than the other two, which indicates it is beneficial to operate the energy facility following the optimal control strategies. The storage problem was further extended to a 3D problem by including one more stochastic variable, like stochastic demand, volatility, convenience yield or another energy commodity price. The 3D problem including stochastic demand was solved as an example and could be easily adapted for other stochastic variables with appropriate boundary conditions. The 3D computation results were consistent with the 2D’s. The numerical convergence order showed the 3D computation results were convergent. Besides, the storage model can be used to solve general commodity storage optimization problems.
5.2 Future directions

5.2.1 Using RHC method

For the storage problem, RHC method can also be used as stated in section 1.2.2.2. RHC is an advanced method of process control used in many areas. For example, Deossa et al. used RHC and Hierarchical Control techniques for portfolio management in energy markets [Deossa2012]. Dunbar et al. used RHC algorithm for distributed control of a platoon of vehicles [Deossa2012]. This method requires to replace all uncertain quantities over the time interval \([t, T]\) with their current estimates using information available at time \(t\) and to solve the optimization problem at each time step. As for the storage problem, the estimated profit \(\hat{P}\) the utility owner/operators can make at each time step \(t\) is

\[
\hat{P}(S,I,\tau) = e^{-r(\tau-t)} \left[ (\hat{u}(\tau) - D)\hat{S}(\tau) + y\hat{I}(\tau) \right],
\]

where the variables with heads on representing estimated values. The optimization problem can be written as

\[
\max \sum_{\tau=t}^{T} \hat{P}(S,I,\tau;\hat{u})
\]

subject to

\[
\hat{I}(\tau+1) = \hat{I}(\tau) - \hat{u}(\tau)d\tau,
\]

\[-U \leq \hat{u} \leq U, \tau = t, t+1, ..., T ,\]

\[0 \leq I(\tau) \leq I_{\text{max}}, \tau = t, t+1, ..., T+1 ,\]

\[\hat{I}(\tau) = I(t).\]
In this method, a proper method to estimate the future prices is needed. By using RHC, the key point of the problem switches from solving a complicated PDE to searching an appropriate model for future prices, which can be an advantage. Another advantage of using RHC is that an automatic code generation software package called CVXGEN may be used. It is developed by Mattingley and Boyd. Detailed description of the software can be found in [Mattingley2009, Mattingley2010a, Mattingley2010b, Mattingley2011]. CVXGEN code takes much less time to solve at each time step than CVX. Computational time is expected greatly decreased by using RHC method with CVXGEN compared with trying to find a better numerical scheme to solve the corresponding PDE in PDE method.

5.2.2 Inclusion of transaction cost

For the processes that produce and consume commodity goods through chemical and physical transformations, transaction cost is important to be considered. For the hedging operation process, break the interval of operations \([t_0, t_K]\) into \(K-1\) discrete subintervals as shown in figure 5.1. There are two kinds of variables involved, the ones with point values at \(t_k\) (\(k=0,1,...,K\)), e.g. \(S(t_k)\), and those with interval values in the \(k^{th}\) interval (\(k=0,1,...,K-1\)), e.g. \(\Theta(k)\). Detailed descriptions about the point and interval variables are listed in table 5.1 and 5.2.
Figure 5.1: Operation intervals and variables

### TABLE 5.1

VARIABLES WITH POINT VALUES

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S(t_k)$</td>
<td>Commodity price at $t_k$</td>
</tr>
<tr>
<td>$W'(k)$</td>
<td>Portfolio value at the very beginning of the $k^{th}$ interval</td>
</tr>
<tr>
<td>$W''(k)$</td>
<td>Portfolio value at the very end of the $k^{th}$ interval</td>
</tr>
<tr>
<td>$L(t_k)$</td>
<td>Cost incurred at the very end of $k^{th}$ interval</td>
</tr>
<tr>
<td>$C(t_k)$</td>
<td>Net cash flow at the very end of $k^{th}$ interval</td>
</tr>
</tbody>
</table>
TABLE 5.2

VARIABLES WITH INTERVAL VALUES

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta(k) )</td>
<td>Hedging strategy in the ( k^{th} ) interval</td>
</tr>
<tr>
<td>( q(k) )</td>
<td>Operation strategy in the ( k^{th} ) interval</td>
</tr>
</tbody>
</table>

The operations are as follows:

1. Obtain current price information \( S_{NG}(t_k) \), \( S_{QL}(t_k) \), and \( S_C(t_k) \) for natural gas(NG), coal(QL) and cash(C), respectively, where \( k=0,1,...,K-1 \).

2. Update the value of the hedging portfolio carried over from \( t_k \) denoted by \( W^+(k) \), where

\[
W^+(k) = \theta_{NG}(k)S_{NG}(t_{k+1}) + \theta_{QL}(k)S_{QL}(t_{k+1}) + \theta_C(k)S_C(t_{k+1})
\]

for \( k=0,...,K-1 \). Variables with index \( K \) are treated 0.

3. Solve a stochastic planning problem to update values for the feed rates \( q_{NG}(k) \) and \( q_{QL}(k) \) and the composition of the hedging portfolio \( \theta_{NG}(k) \), \( \theta_{QL}(k) \) and \( \theta_C(k) \) in the interval \( (t_k, t_{k+1}] \) (\( k=0, 1,..., K-1 \)), subject to

\[
q_{NG}(k), q_{QL}(k) \geq 0, \\
q_{NG}(k) + q_{QL}(k) = 1.
\]
Additional constraints may be imposed on the process or hedging portfolio, such as no shorting (i.e. \( \theta_{\text{NG}}(k) \geq 0, \theta_{\text{QL}}(k) \geq 0 \)), no borrowing (i.e. \( \theta_c(k) \geq 0 \)), or further constraints on allowable feed rates and feed compositions.

4. Update the value of the hedging portfolio following the hedging transactions denoted by \( W(k) \) for \( k=0,1, \ldots, K \), given by

\[
W^-(k) = \theta_{\text{NG}}(k)S_{\text{NG}}(t_k) + \theta_{\text{QL}}(k)S_{\text{QL}}(t_k) + \theta_c(k)S_c(t_k).
\]

The initial value \( W(t_0) \) of the hedging portfolio is a key decision variable. At \( t=K \), we treat \( W(t_k)=0 \).

5. Purchase raw materials for hedging and processing for the interval \( [t_k, t_{k+1}] \) (\( k=0, \ldots, K-1 \)) and pay any transaction costs. The cost incurred at time \( t_k \) is

\[
L(t_k) = [S_{\text{NG}}(t_k)H_{\text{NG}}q_{\text{NG}}(k) + S_{\text{QL}}(t_k)H_{\text{QL}}q_{\text{QL}}(k)]\delta t - \theta_{\text{NG}}(k)y_{\text{NG}}\delta t - \theta_{\text{QL}}(k)y_{\text{QL}}\delta t
+ \tau_{\text{NG}} |\theta_{\text{NG}}(k) - \theta_{\text{NG}}(k-1) + H_{\text{NG}}q_{\text{NG}}(k)\delta t|
+ \tau_{\text{QL}} |\theta_{\text{QL}}(k) - \theta_{\text{QL}}(k-1) + H_{\text{QL}}q_{\text{QL}}(k)\delta t|,
\]

where \( \delta t=t_{k+1}-t_k \). All of the variables whose index goes below 0 are treated to be 0. Parameters \( \tau_{\text{NG}} \) and \( \tau_{\text{QL}} \) are transaction costs per unit of NG and QL, respectively. It is assumed that no additional costs are incurred when raw materials are withdrawn from physical storage and consumed by the process. The hedging operations can result in buying or selling of NG and QL; the absolute value functions measure the magnitude of

---

\(^2\) In finance, shorting is the practice of selling financial instruments that are not currently owned, and subsequently repurchasing them.
the total transaction in NG and QL at time $t_k$. If the costs of the buying and selling transactions are different then the cost becomes

$$
L(t_k) = \left[ S_{NG}(t_k) H_{NG} q_{NG}(k) + S_{QL}(t_k) H_{QL} q_{QL}(k) \right] \delta t \\
- \theta_{NG}(k) y_{NG} \delta t - \theta_{QL}(k) y_{QL} \delta t \\
+ \tau_{NG}^{buy} \left| \theta_{NG}(k) - \theta_{NG}(k-1) + H_{NG} q_{NG}(k) \right| \delta t \\
+ \tau_{NG}^{sell} \left| \theta_{NG}(k-1) - \theta_{NG}(k) - H_{NG} q_{NG}(k) \right| \delta t \\
+ \tau_{QL}^{buy} \left| \theta_{QL}(k) - \theta_{QL}(k-1) + H_{QL} q_{QL}(k) \right| \delta t \\
+ \tau_{QL}^{sell} \left| \theta_{QL}(k-1) - \theta_{QL}(k) + H_{QL} q_{QL}(k) \right| \delta t.
$$

where $[\xi]^+ = \max(0, \xi)$ and parameters $\tau^{buy}$ and $\tau^{sell}$ distinguish between the buying and selling transactions.

6. Fund the net cash requirement $C(t_k)$ of the combined hedging and process operations at time $t_k$ for $k=0, 1, \ldots, K-1$

$$
C(t_k) = \frac{L(t_k) - \left( W^+(t_k) - W^-(t_k) \right)}{\text{cost/ hedging income}}.
$$

For example, if we don’t consider transaction cost for $k=0, 1, \ldots, K-1$, then

$$
C(t_k) = \left[ S_{NG}(t_k) H_{NG} q_{NG}(k) + S_{QL}(t_k) H_{QL} q_{QL}(k) \right] \delta t \\
- \left( \theta_{NG}(k) dS_{NG}(t_k) + \theta_{QL}(k) dS_{QL}(t_k) + \theta_{c}(k) dS_{C}(t_k) \right) \\
- \theta_{NG}(k) y_{NG} \delta t - \theta_{QL}(k) y_{QL} \delta t.
$$

If $dS_{C}(t_k) = rS_{C}(t_k) \delta t$,

$$
C(t_k) = \left[ S_{NG}(t_k) H_{NG} q_{NG}(k) + S_{QL}(t_k) H_{QL} q_{QL}(k) \right] \delta t \\
- \left( \theta_{NG}(k) dS_{NG}(t_k) + \theta_{QL}(k) dS_{QL}(t_k) + r(W^-(k) - \theta_{NG}(k) dS_{NG}(t_k) - \theta_{QL}(k) dS_{QL}(t_k)) \right) \\
- \theta_{NG}(k) y_{NG} \delta t - \theta_{QL}(k) y_{QL} \delta t.
$$

If the utility uses the minimum cost and the net cash

$$
C(t_k) = -dW^-(k) = W^-(k) - W^+(k).
$$
Suppose the hedging decisions happen really quickly, then $W^+(k) = W^-(k + 1)$,

$$-dW^-(k) = \min[S_{NG}(t_k)H_{NG}q_{NG}(k) + S_{QL}(t_k)H_{QL}q_{QL}(k)]\delta t$$

$$\quad -\left(\theta_{NG}(k)dS_{NG}(t_k) + \theta_{QL}(k)dS_{QL}(t_k) + r(W^-(k) - \theta_{NG}(k)dS_{NG}(t_k) - \theta_{QL}(k)dS_{QL}(t_k))\right)$$

$$\quad -\theta_{NG}(k)y_{NG}\delta t - \theta_{QL}(k)y_{QL}\delta t$$

The cost of operations $V$ is the present value of all cash flows required to finance and maintain the hedging portfolio, i.e.,

$$V = \sum_{k=0}^{K-1} e^{-r(t_k-t_0)}C(t_k).$$

$V$ is a random variable due to the stochastic process governing the prices of NG and QL. The expected cost of operations is

$$E[V] = \sum_{k=0}^{K-1} e^{-r(t_k-t_0)}E[C(t_k)].$$

The problem of minimizing expected cost has a 'pay me now' or 'pay me later' aspect. The process operator can attempt to finance operations upfront by putting money into a hedging portfolio $W^+(t_0)$. Or instead, the operation could choose to pay cost $C(t_k)$ as they are incurred. The cash component of the hedging portfolio is invested (or borrowed) with a rate of return equal to the discount rate, so the value of $V$ is indifferent to this choice.

For an operator interested in minimizing the expected cost of operations the problem is

$$\min_{W^+(t_0)q_{NG}(t_k)q_{QL}(t_k)\theta_{NG}(t_k)\theta_{QL}(t_k)} E[V]$$

subject to process constraints.
and joint process and hedging constraints for \( k=0, \ldots, K-1 \)

\[
q_{NG}(k), q_{QL}(k) \geq 0, \quad k = 0, \ldots, K-1
\]

\[
q_{NG}(k) + q_{QL}(k) = 1, \quad k = 0, \ldots, K-1
\]

\[
E[V] \geq W^-(0),
\]

\[
W^-(k) = \theta_{NG}(k)S_{NG}(t_k) + \theta_{QL}(k)S_{QL}(t_k) + \theta_c(k)S_c(t_k),
\]

\[
W^+(k) = \theta_{NG}(k)S_{NG}(t_{k+1}) + \theta_{QL}(k)S_{QL}(t_{k+1}) + \theta_c(k)S_c(t_{k+1}),
\]

\[
L(t_k) = [S_{NG}(t_k)H_{NG}q_{NG}(k) + S_{QL}(t_k)H_{QL}q_{QL}(k)]\delta t
\]

\[
-\theta_{NG}(k)y_{NG}\delta t - \theta_{QL}(k)y_{QL}\delta t
\]

\[
+ \tau_{NG} |\theta_{NG}(k) - \theta_{NG}(k-1) + H_{NG}q_{NG}(k)|\delta t |
\]

\[
+ \tau_{QL} |\theta_{QL}(k) - \theta_{QL}(k-1) + H_{QL}q_{QL}(k)|\delta t |
\]

\[
\theta_{C}(t_{-1}) = 0,
\]

\[
\theta_{QL}(t_{-1}) = 0,
\]

\[
\theta_{NG}(t_{-1}) = 0,
\]

\[
C(t_k) = L(t_k) - (W^+(k) - W^-(k)),
\]

\[
W^+(K-1) = 0.
\]
APPENDIX A:

MATLAB CODE

A.1 Code for energy swap valuation and validation

clc
clear all

%% GBM Price Model
nMC=50; % # of price chains (Monte Carlo simulation times)
nMonth=3; % # of days
Tr= round(365/12*nMonth);
Ttra= round(252/12*nMonth);

% Standard deviation
sigmaNG = 0.0265*sqrt(Ttra/Tr);
sigmaQL = 0.0145*sqrt(Ttra/Tr);

S0NG=3; % typical initial price of natural gas (NG)
S0QL=60; % typical initial price of coal (QL) (can be S0NG*YNG/YQL)

muNG=0.001*Ttra/Tr; % drift of NG
muQL=0.002*Ttra/Tr; % drift of QL

% correlation
rou = 0.2647;

% time step
dt_vali=1;

% standard deviation
sigma = [sigmaNG^2 rou*sigmaNG*sigmaQL; rou*sigmaNG*sigmaQL ... sigmaQL^2];

% initialization of price
PPNG=zeros(nMC,Tr);
PPQL=zeros(nMC,Tr);

% MC for GMB prices
dW = sqrt(dt_vali)*mvnrnd(zeros(2,1),sigma,nMC);
% PPNG(:,1)=S0NG*(1 + muNG*dt_vali + dW(:,1));
PQL(:,1) = S0QL * (1 + muQL * dt_vali + dW(:,2));
PPNG(:,1) = S0NG;
PPQL(:,1) = S0QL;

\[ t = \text{dt\_vali:dt\_vali:Tr}; \]

for k = 2:length(t)
\[ dW = \sqrt{\text{dt\_vali}} \times \text{mvnrnd(zeros(2,1),sigma,nMC)}; \]
PPNG(:,k) = PPNG(:,k-1) * (1 + muNG * dt_vali + dW(:,1));
PPQL(:,k) = PPQL(:,k-1) * (1 + muQL * dt_vali + dW(:,2));
end

figure(1); clf;

subplot(2,1,1)
plot(t,PPNG)
xlabel('t/day')
ylabel('NG/$/MMBtu')
ax = axis;
axis([ax(1) Tr ax(3) ax(4)]);

subplot(2,1,2)
plot(t,PPQL)
xlabel('t/day')
ylabel('Coal/$/ton')
ax = axis;
axis([ax(1) Tr ax(3) ax(4)]);

figure(2); clf;
subplot(2,4,[1 2]);
hist(PPNG(:,end),sqrt(nMC));
subplot(2,4,[3 4]);
hist(PPQL(:,end),sqrt(nMC));
subplot(2,4,[6 7]);
plot(PPNG(:,end),PPQL(:,end),'.','Markersize',2);
axis 'square';

%% find thetaNG and thetaQL
% Risk free discount rate
r = exp(log(1.1)/365)-1;
% Convenience Yields
yNG=0;
yQL=0.2;
% Heating Rates
YQL=10.5; % Unit (ton/d/MW)
YNG=196.8; % Unit (MMbtu/d /MW)
% time step
dt=0.005;

% grid number and grid size
N=40;
h=1/40;

% NG and QL prices
PNGmax=50;
PNGmin=0;
PQLmax=400;
PQLmin=0;

PgNG=0:h:1;
PNG=length(PgNG);
dPNG=mean(diff(PgNG));

PgQL=0:h:1;
PQL=length(PgQL);
dPQL=mean(diff(PgQL));

PNGV=(PNGmax-
PNGmin) * PgNG + PNGmin;
PQLV=(PQLmax-PQLmin) * PgQL + PQLmin;

NG=PNGV'*ones(1,PQL);
NG2=NG.^2;

QL=ones(PNG,1)*PQLV;
QL2=QL.^2;

% Cost function
C=min(NG*YNG, QL*YQL);

% Initialization
V=zeros(PNG,PQL);
DppNG=zeros(PNG,PQL);
DpNG=zeros(PNG,PQL);
DppQL=zeros(PNG,PQL);
DpQL=zeros(PNG,PQL);
DppNGQL=zeros(PNG,PQL);
thetaNG=zeros(nMC,Tr);
thetaQL=zeros(nMC,Tr);
Va1=zeros(nMC,Tr);

idx=2:PNG-1;
jdx=2:PQL-1;

for m=1:dt:Tr
    % discretization of derivatives for DpNG and DppNG
    % boundary conditions (2nd derivative equals 0 when NG price is very small or large)
    DppNG(1,:)=0;
    DpNG(1,:)=(V(2,:)-V(1,:))/dPNG;
    DppNG(end,:)=0;
    DpNG(end,:)=(V(end,:)-V(end-1,:))/dPNG;

% Initialization
% 2nd derivative w.r.t. price of NG using central approximation
\[ D_{ppNG}(idx,:) = \frac{(V(idx+1,:) - 2*V(idx,:) + V(idx-1,:))}{dPNG^2}; \]

% 1st derivative w.r.t. price of QL using upwind method which % depends on the sign of the coefficient in front of it
\[ D_{pNG}(idx,:) = \frac{(V(idx+1,:) - V(idx,:))}{dPNG}; \]

\[ \text{mu}_{idx} = \text{find}(y_{NG} - r \cdot PNGV(idx) > 0); + 1; \]
if isempty(mu_{idx}) == 1
\[ D_{pNG}(mu_{idx},:) = (V(mu_{idx},:) - V(mu_{idx}-1,:)) / dPNG; \]
end

% discretization of derivatives for DpQL and DppQL
% boundary conditions (2nd derivative equations 0 when QL price is very small or large)
\[ D_{ppQL}(1,:) = 0; \]
\[ D_{pQL}(1,:) = (V(:,2) - V(:,1)) / dPQL; \]
\[ D_{ppQL}(end,:) = 0; \]
\[ D_{pQL}(end,:) = (V(:,end) - V(:,end-1)) / dPQL; \]

% 2nd derivative wrt price of QL using central approximation
\[ D_{ppQL}(jdx,:) = (V(:,jdx+1) - 2*V(:,jdx) + V(:,jdx-1)) / dPQL^2; \]

% 1st derivative wrt price of QL using upwind method which % depends on the sign of the coefficient in front of it
\[ D_{pQL}(jdx,:) = (V(:,jdx+1) - V(:,jdx)) / dPQL; \]

\[ \text{mu2}_{idx} = \text{find}(y_{QL} - r \cdot PQLV(jdx) > 0); + 1; \]
if isempty(mu2_{idx}) == 1
\[ D_{pQL}(:,mu2_{idx}) = (V(:,mu2_{idx}) - V(:,mu2_{idx}-1)) / dPQL; \]
end

% discretization for mixed derivatives with central approximation
\[ D_{ppNGQL}(1,1) = (V(2,2) - V(1,2) - V(2,1) + V(1,1)) / dPNG/dPQL; \]
\[ D_{ppNGQL}(end,1) = (V(end,2) - V(end-1,2) - V(end,1) + V(end-1,1)) / dPNG/dPQL; \]
\[ D_{ppNGQL}(1,end) = (V(2,end) - V(2,end-1) + V(1,end) - V(1,end-1)) / dPNG/dPQL; \]
\[ D_{ppNGQL}(end,end) = (V(end,end) - V(end-1,end) - V(end,end-1) + V(end-1,end-1)) / dPNG/dPQL; \]

\[ D_{ppNGQL}(1,jdx) = (V(2,jdx+1) - V(1,jdx+1) - V(2,jdx-1) + V(1,jdx-1)) / dPNG/2/dPQL; \]

\[ D_{ppNGQL}(idx,1) = (V(idx+1,2) - V(idx-1,2) - V(idx+1,1) + V(idx-1,1)) / dPNG/2/dPQL; \]
\[ D_{ppNGQL}(idx,jdx) = (V(idx+1,jdx+1) - V(idx-1,jdx+1) - V(idx+1,jdx-1) + V(idx-1,jdx-1)) / 4/dPNG/dPQL; \]
\[ D_{ppNGQL}(idx,end) = (V(idx+1,end) - V(idx-1,end) - V(idx+1,end-1) + V(idx-1,end-1)) / dPNG/2/dPQL; \]
\[ D_{ppNGQL}(end,jdx) = \frac{V(end,jdx+1) - V(end-1,jdx+1) - V(end,jdx-1) + V(end-1,jdx-1)}{dPNG/2/dPQL}; \]

% find the hedging decisions \theta
% iterate V at next time step
if m==round(m)
    \text{Va1(:,m)} = \text{interp2(PQLV,PNGV,V,PPQL(:,Tr+1-m),PPNG(:,Tr+1-m));}
    \text{thetaNG(:,m)} = \text{interp2(PQLV,PNGV,DpNG/(PNGmax-PNGmin),PPQL(:,Tr+1-m),PPNG(:,Tr+1-m));}
    \text{thetaQL(:,m)} = \text{interp2(PQLV,PNGV,DpQL/(PQLmax-PQLmin),PPQL(:,Tr+1-m),PPNG(:,Tr+1-m));}
end

V=V+dt*\left(\frac{1}{2}\sigma_{NG}^2*NG^2*D_{ppNG}(PNGmax-PNGmin)^2+\text{rou}\sigma_{NG}\sigma_{QL}NG*QL*D_{ppNGQL}(PNGmax-PNGmin)/(PQLmax-PQLmin)+\frac{1}{2}\sigma_{QL}^2*QL^2*D_{ppQL}(PQLmax-PQLmin)^2+(r*NG-yNG)*D_{ppNG}(PNGmax-PNGmin)+(r*QL-yQL)*D_{ppQL}(PQLmax-PQLmin)-r*V+C\right);

%%% validation ---calculate V of self-financing portfolio
%%% reverse the hedging decisions theta to make it go forward
thetaNG=thetaNG(:,Tr:-1:1);
thetaQL=thetaQL(:,Tr:-1:1);

% intialize new V
\text{Va} = \text{zeros(nMC,Tr)};
\text{Va(:,1)} = \text{Va1(:,Tr)};

% calculate forward the hedging portfolio value using valuation function
for k=1:Tr-1
    \text{Va(:,k+1)} = \text{Va(:,k)} + r*(\text{Va(:,k)}-\text{thetaNG(:,k)}*PPNG(:,k)-\text{thetaQL(:,k)}*PPQL(:,k))\cdot dt_vali+\text{thetaNG(:,k)}*(PPNG(:,k+1)-PPNG(:,k))+\text{thetaQL(:,k)}*(PPQL(:,k+1)-PPQL(:,k)) - ...
        min(PPNG(:,k)*YNG,PPQL(:,k)*YQL)*dt_vali+
        \text{thetaNG(:,k)}*yNG*\text{dt}+\text{thetaQL(:,k)}*yQL*\text{dt_vali};
end

%%% how much cash in the hedge
\text{rtncash} = \text{Va}-\text{thetaNG}.*\text{PPNG}-\text{thetaQL}.*\text{PPQL};

%%% plot
subplot(3,2,1)
plot(t,Va)
xlabel('Time/day')
ylabel('V/$/MW')

subplot(3,2,2)
plot(t,thetaNG)
xlabel('Time/day')
ylabel('ThetaNG/MMBtu/MW')

subplot(3,2,3)
plot(t,thetaQL)
xlabel('Time/day')
ylabel('ThetaQL/ton/MW')

subplot(3,2,4)
plot(t,PPNG.*thetaNG)
xlabel('Time/day')
ylabel('PNgThetaNg')

subplot(3,2,5)
plot(t,PPQL.*thetaQL)
xlabel('Time/day')
ylabel('PQLThetaQL')

figure
plot(t,rtncash,'g',t,PPNG.*thetaNG,'b',t,Va,'r')
xlabel('Time/day')
ylabel('V/$/MW')

A.2 Code for 2D storage problem

clc
clear all

%% Validation information
N=1; % # of price chains (Monte Carlo simulation times)
Tr=63; % # of days
P0=50; % initial value for price
muP=0.25; % drift
sigmaP = 0.2; % sigma in price model
MeanR_P=100;
% time step
Vali_dt=.1;
t = 1:Vali_dt:Tr;
len_t=length(t);

Vali_Pg=zeros(N,len_t);
Vali_Pg(:,1)=P0;
for k = 2:len_t
    dW = sqrt(Vali_dt)*randn(N,1);
    Vali_Pg(:,k) = Vali_Pg(:,k-1)+ muP*(MeanR_P-Vali_Pg(:,k-1))*Vali_dt +
                   sigmaP*Vali_Pg(:,k-1).*dW(:,1);
end
plot(t,Vali_Pg)
% if starting from diferent P0
% Vali_Pg=zeros(N,len_t+1);
% Vali_Pg(:,1)=P0;
% for k = 2:len_t+1
%     dW = sqrt(Vali_dt)*randn(N,1);
%     Vali_Pg(:,k) = Vali_Pg(:,k-1) + muP*(100-Vali_Pg(:,k-1))*Vali_dt +
%                    sigmaP*Vali_Pg(:,k-1).*dW(:,1);
% end
% Vali_Pg=Vali_Pg(:,2:end);

%% Parameters
r = 0.1/365; % discounted rate
sigma=sigmaP;
y=0.2; % convenience yield
D=0; % demand
U=100; % maximum control
dt=0.0005; % time step
Nod=20; % no of nodes
h=1/Nod; % grid step

%% Computation region
Pmin=0; % minimum price
Pmax=250; % maximum price
Imin=0; % minimum inventory
Imax=1500; % maximum inventory
I0=1000; % initial inventory

%% find optimal control and valuation results
Pg=0:h:1;
P=length(Pg);
dP=mean(diff(Pg));

Ig=0:h:1;
I=length(Ig);
dI=mean(diff(Ig));

Pg1=(Pmax-Pmin)*Pg+Pmin;
Ig1=(Imax-Imin)*Ig+Imin;

idx=2:P-1;
jdx=2:I-1;

Cmin=-U;
Cmax=U;
contrl=[Cmin,0,Cmax];
N1=Pg1'*ones(1,I);
M=N1.^2;

II=ones(P,1)*Ig1;
mu=muP*(MeanR_P-N1);

%% Initialization
V=zeros(P,1);
Vali_V=zeros(N,len_t);
Vali_Cn=zeros(N,len_t);
Vali_Ig=ones(N,len_t+1)*I0;
B=zeros(len_t,I); % B represents switching line
Cn=zeros(P,I); % initialize control
Vali_Cn1=zeros(P,I,len_t);
Vali_V1=Vali_Cn1;
Dpp=zeros(P,I); % initialize Vpp
Dp=zeros(P,I); % initialize Vp
DI1=zeros(P,I); % initialize V1 (upwind up)
DI2=zeros(P,I); % initialize V1 (upwind down)
Q=zeros(P,I);  % initialize optimal part -c(i,j)*VI(i,j)+c(i,j)*P(i)
Opt_zero=zeros(P,I);

count=1;

for k=1:dt:Tr
% build Dpp
Dpp(1,:)=0; % Boundary condition
Dp(1,:)=(V(2,:)-V(1,:))/dP;
Dpp(idx,:)=(V(idx+1,:)-2*V(idx,:)+V(idx-1,:))/dP^2;
Dp(idx,:)=(V(idx,:)-V(idx-1,:))/dP;
mu_idx=find(mu(idx,1)>0)+1;
Dp(mu_idx,:)=(V(mu_idx+1,:)-V(mu_idx,:))/dP;
Dpp(P,:)=0; % Boundary condition
Dp(P,:)=(V(P,:)-V(P-1,:))/dP;

% build DI (both upwind up and upwind down)
DI1(:,1)=(-V(:,1)+V(:,2))/dI; % Boundary condition
DI2(:,1)=DI1(:,1); % Boundary condition
DI1(:,jdx)=(V(:,jdx+1)-V(:,jdx))/dI;
DI2(:,jdx)=(-V(:,jdx-1)+V(:,jdx))/dI;
DI1(:,I)=(V(:,I)-V(:,I-1))/dI; % Boundary condition
DI2(:,I)=DI1(:,I); % Boundary condition

Opt_min = Cmin*(N1-DI1/(Imax-Imin));
Opt_max = Cmax*(N1-DI2/(Imax-Imin));

[Q(:,1),ind]= max(cat(3,Opt_min(:,1),Opt_zero(:,1)),[]),3);
Cn(:,1)=contrl(ind);

[Q(:,jdx),ind]= max(cat(3,Opt_min(:,jdx),Opt_zero(:,jdx),Opt_max(:,jdx)),[]),3);
Cn(:,jdx)=contrl(ind);

[Q(:,I),ind]= max(cat(3,Opt_zero(:,I), Opt_max(:,I)),[]),3);
Cn(:,I)=contrl(ind+1);

% To iterate V at next time step based on the V at the current time step
V=V+dt*(1/2*sigma^2*M.*Dpp/(Pmax-Pmin)^2+mu.*Dp/(Pmax-Pmin)-D*N1+y.*H.*y*V+Q);
% %
if k==t(count)
   Vali_V1(:,:,count)=V;
   Vali_Cn1(:,:,count)=Cn;
   for j=1:I
      B(count,j)=Pg1(min(find(Cn(:,j)>=0)));
   end
   % for i=1:P
   %      B(count,i)=Ig1(min(find(Cn(i,:)>=0)));
   % end
   count=count+1;
end
% plot results
subplot(2,1,1)
surf(Ig1,Pg1,V)
xlabel('I(bbl)')
ylabel('S($/bbl)')
zlabel('V(\$)')
subplot(2,1,2)
surf(Ig1,Pg1,Cn)
xlabel('I/bbl')
ylabel('S($/bbl)')
zlabel('u(bbl/d)')
% drawnow
end
for m=1:len_t
   Vali_V(:,m)=interp2(Ig,Pg,Vali_V1(:,:,len_t+1-m),(Vali_Ig(:,(m))-Imin)/(Imax-Imin),(Vali_Pg(:,m)-Pmin)/(Pmax-Pmin),'nearest');
   Vali_Cn(:,m)=interp2(Ig,Pg,Vali_Cn1(:,:,len_t+1-m),(Vali_Ig(:,(m))-Imin)/(Imax-Imin),(Vali_Pg(:,m)-Pmin)/(Pmax-Pmin),'nearest');
   Vali_Ig(:,m+1)=Vali_Ig(:,m)-Vali_Cn(:,m)*Vali_dt;
end
%% video plot
video = VideoWriter('ValidationControlStrategy.avi');
video.FrameRate = 15;
open(video);
hf = figure ;clf;
set(hf,'Position',[100 100 1000 500]);
for m=1:len_t
   %plot(Vali_Ig(:,1:m),Vali_Pg(:,1:m),'r-','Markersize',20)
   plot(Vali_Ig(:,m),Vali_Pg(:,m),'r.','Markersize',20)
   hold on
   plot(Ig1,B(len_t+1-m,:))
   hold off
   xlabel('I/bbl')
   ylabel('P/$/bbl')
   axis([0 2000 0 180])
drawnow
frame = getframe(hf);
writeVideo(video[frame]);
end
close(video);

hist(Vali_V(:,end),30)

figure(1);clf;
subplot(3,1,1)
plot(t,Vali_Pg)
xlabel('t/day')
ylabel('P/$/barrel')
ax = axis;
axis([ax(1) Tr ax(3) ax(4)]);
subplot(3,1,2)
plot(t,Vali_Ig(end)
xlabel('t/day')
ylabel('I/barrel')
ax = axis;
axis([ax(1) Tr ax(3) ax(4)]);
subplot(3,1,3)
plot(t,Vali_Cn)
xlabel('t/day')
ylabel('Cn/barrel/day')
ax = axis;
axis([ax(1) Tr ax(3) ax(4)]);

figure
% data=smooth3(Vali_Cn1); % smooth the value first
% isosurface(lg1,Pg1,t,data(:,end-1:1),0);
% isosurface(lg1,Pg1,t,Vali_Cn1(:,end-1:1),0);
hold on
plot3(Vali_Ig(1:len_t),Vali_Pg(1:len_t),t,'LineWidth',2)
grid on
hold on
plot3(real_Ig(1:len_t),Vali_Pg(1:len_t),t,'LineWidth',2)
%plot3(Pre_Ig(1:len_t)-[0 mean(diff(lg1))/2*ones(1,len_t-1)],Vali_Pg(1:len_t),t)
hold off
ylabel('S/$/bbl')
xlabel('I/barrel')
zlabel('Real t/d')
title ('Control=0')
A.3 Code for 3D storage problem

%% solve 3D HJB equation

%% Vt+1/2*(b)^2*Vpp+ a*Vp+1/2*q^2*Vdd+p*Vd+rho*b*q*Vpd-r*D*S+y*I+max(u(P-Vt))=0
%% dP=a*dt+b*dZ, where a=?(-P), b=sigmaP*P
%% dD=p*dt+q*dZ, where p=?.(P)*D, q=sigmaD*D

%% notice
% grid size N can't be too big, or else it will be out of memory.

clear all

%% Parameters

r = 0.1/365; % risk-free discounted rate
sigmaP=0.2;
sigmaD=0.25;
rho=.0023;
y=.2; % convenience yield
U=100; % control

T=10; % caculation time
dt=.0001; % time step

N=80; % grid number
h=1/N; %grid size
t=0:dt:T;

% calculation range
Pmin=0;
Pmax=150;

Imin=0;
Imax  = 1500;

Dmin=0;
Dmax=500;

%%

Pg=0:h:1; % range of price
P=length(Pg);
dP=mean(diff(Pg));

Ig=0:h:1; % range of inventory
I=length(Ig);
dl=mean(diff(Ig));

Dg=0:h:1; % range of demand
D=length(Dg);
dD=mean(diff(Dg));

Cmin=-U; % the minimum control
Cmax=U; % the maximum control
contrl=[Cmin,0,Cmax];

% to get matrix to calculate a,p and b^2=sgimaP^2*Pg(i)^2, % q^2=sgimaD^2*Dg(m)^2

N1=ones(P,I,D);
for i=1:P
   N1(i,:,:)=N1(i,:,:)*((Pmax-Pmin)*Pg(i)+Pmin);
end

N2 =ones(P,I,D);
for j=1:I
   N2 (:,j,:)=N2(:,j,:)*((Imax-Imin)*Ig(j)+Imin);
end

N3=ones(P,I,D);
for m=1:D
   N3(:,:,m)=N3(:,:,m)*((Dmax-Dmin)*Dg(m)+Dmin);
end

Pg1=(Pmax-Pmin)*Pg+Pmin;
Ig1=(Imax-Imin)*Ig + Imin;

%%
a=0.25*(100-N1);
b=sgimaP*N1;
p=0.50*(50-N3).*N3;
q=sgimaD*N3;

V=zeros(P,I,D); % V(T) the initial plane for V
Cn=zeros(P,I,D); % initialize control
Dpp=zeros(P,I,D); % initialize Vpp
Dp=zeros(P,I,D); % initialize Vp
Ddd=zeros(P,I,D); % initialize
Dd=zeros(P,I,D);
DpDd=zeros(P,I,D);
DI1=zeros(P,I,D); % initialize VI (upwind up)
DI2=zeros(P,I,D); % initialize VI (upwind down)
Q=zeros(P,I,D);  % initialize optimal part
Opt_zero = zeros(P,I,D);
idx = 2:(P-1);
jdx = 2:(I-1);
mdx = 2:(D-1);
%%% for the situation selling out the leftovers
%
% for m=1:D
% V(:, m)=((Pmax-Pmin)*Pg+Pmin)^a*(((Imax-Imin)*Ig+Imin);
% end

%%% calculate derivatives
for k=1:length(t)-1

    % build Dpp
    Dpp(1,:,:)=0; % Boundary condition
    Dp(1,:,:)=(V(2,:,:)-V(1,:,:))/dP;
    Dpp(idx,:,:)=(V(idx+1,:,:)-2*V(idx,:,:)+V(idx-1,:,:))/dP^2;
    Dp(idx,:,:)=(V(idx,:,:)-V(idx-1,:,:))/dP;
    a_idx=find(a(idx,1,1)>0)+1;
    Dp(a_idx,:,:)=(V(a_idx+1,:,:)-V(a_idx,:,:))/dP;
    % for i=2:(P-1)
    % if a(i,1,1)>0
    %     Dp(i,:,:)=(V(i+1,:,:)-V(i,:,:))/dP;
    % else
    %     Dp(i,:,:)=(V(i,:,:)-V(i-1,:,:))/dP;
    % end
    % end
    Dpp(P,:,:)=0; % Boundary condition
    Dp(P,:,:)=(V(P,:,:)-V(P-1,:,:))/dP;

    % build Ddd
    Ddd(:,1,:)=0; % Boundary condition
    Dd(:,1,:)=(V(:,2,:)-V(:,1,:))/dD;
    Ddd(:,mdx,:)=(V(:,mdx+1,:)-2*V(:,mdx,:)+V(:,mdx-1,:))/dD^2;
    Dd(:,mdx,:)=(V(:,mdx,:)-V(:,mdx-1,:))/dD;
    p_idx=find(p(1,1,mdx)>0)+1;
    Dd(:,p_idx,:)=V(:,p_idx+1,:)-V(:,p_idx,:))/dD;
    % for m=2:(D-1)
    % if p(1,1,m)>0
    %     Dd(:,m,:)=(V(:,m+1,:)-V(:,m,:))/dD;
    % else
    %     Dd(:,m,:)=(V(:,m,:)-V(:,m-1,:))/dD;
    % end
    % end
    Ddd(:,D,:)=0; % Boundary condition
    Dd(:,D,:)=(V(:,D,:)-V(:,D-1,:))/dD;

    % build DI (both upwind up and upwind down)
    DI1(:,1,:)=(-V(:,1,:)+V(:,2,:))/dI; % Boundary condition
    DI2(:,1,:)= DI1(:,1,:); % Boundary condition
DI1(:,jdx,:)=(V(:,jdx+1,:) - V(:,jdx,:))/dI;
DI2(:,jdx,:)=(V(:,jdx,:) - V(:,jdx-1,:))/dI;
DI1(:,I,:)=(V(:,I,:) - V(:,I-1,:))/dI;  % Boundary condition
DI2(:,I,:)=DI1(:,I,:);  % Boundary condition

% build DpDd
DpDd(1,:,1)=(V(2,:,2) - V(1,:,2) - V(2,:,1) + V(1,:,1))/dP/dD;
DpDd(end,:,1)=(V(end,:,2) - V(end-1,:,2) - V(end,:,1) + V(end-1,:,1))/dP/dD;
DpDd(1,:,end)=(V(2,:,end) - V(1,:,end) - V(2,:,end-1) + V(1,:,end-1))/dP/dD;
DpDd(end,:,end)=(V(end,:,end) - V(end-1,:,end) - V(end,:,end-1) + V(end-1,:,end-1))/dP/dD;
DpDd(idx,:,1)=(V(idx+1,:,2) - V(idx-1,:,2) - V(idx+1,:,1) + V(idx-1,:,1))/dP/2/dD;
DpDd(idx,:,1)=(V(idx+1,:,end) - V(idx-1,:,end) - V(idx+1,:,end-1) + V(idx-1,:,end-1))/dP/2/dD;
DpDd(1,:,mdx)=(V(2,:,mdx+1) - V(1,:,mdx+1) - V(2,:,mdx) + V(1,:,mdx))/dP/2/dD;
DpDd(end,:,mdx)=(V(end,:,mdx+1) - V(end-1,:,mdx+1) - V(end,:,mdx) + V(end-1,:,mdx))/dP/2/dD;

% for m=2:D-1
%    DpDd(idx,:,m)=(V(idx+1,:,m+1) - V(idx-1,:,m+1) - V(idx+1,:,m-1) + V(idx-1,:,m-1))/4/dP/dD;
% end
DpDd(idx,:,mdx)=(V(idx+1,:,mdx+1) - V(idx-1,:,mdx+1) - V(idx+1,:,mdx) + V(idx-1,:,mdx))/4/dP/dD;

% simplified code for obtaining control strategy
Opt_min = Cmin*(N1-DI1/(Imax-Imin));
Opt_max = Cmax*(N1-DI2/(Imax-Imin));

\[ Q(:,1,:),ind\] = max(cat(4,Opt_min(:,1,:),Opt_zero(:,1,:)),[],4);
Cn(:,1,:)=ctrl(ind);

\[ Q(i,j,:),ind\] = max(cat(4,Opt_min,Opt_zero,Opt_max),[],4);
\[ Q(:,jdx,:),ind\] = max(cat(4,Opt_min(:,jdx,:),Opt_zero(:,jdx,:),Opt_max(:,jdx,:)),[],4);
Cn(:,jdx,:)=ctrl(ind);

\[ Q(:,1,:),ind\] = max(cat(4,Opt_zero(:,1,:), Opt_max(:,1,:)),[],4);
Cn(:,1,:)=ctrl(ind+1);

% To iterate V at next time step based on the V at the current time step
V = V + dt*(1/2*b.^2.*Dpp/(Pmax-Pmin)^2 + a.*Dp/(Pmax-Pmin) + 1/2*q.^2.*Ddd/(Dmax-Dmin)^2 + p.*Dd/(Dmax-Dmin) + rho*b.*q.*DpDd/(Dmax-Dmin)/(Pmax-Pmin)-N1.*N3-r*V+y*N2+Q);

% Plot figures
subplot(2,1,1)
title(num2str(k*dt,'%6.4f'))
surf(Ig1,Pg1,V(:,;1))
xlabel('I (bbl)')
ylabel('S ($/bbl)')
zlabel('V ($)')

subplot(2,1,2)
surf(Ig1,Pg1,Cn(:,:,1))
xlabel('I (bbl)')
ylabel('S ($/bbl)')
zlabel('u (bbl/d)')

drawnow
end


74. Eduardo Schwartz, The real options approach to valuation: challenges and opportunities, UCLA Adnerson School, 2012 Alberta Finance Institute Conference


